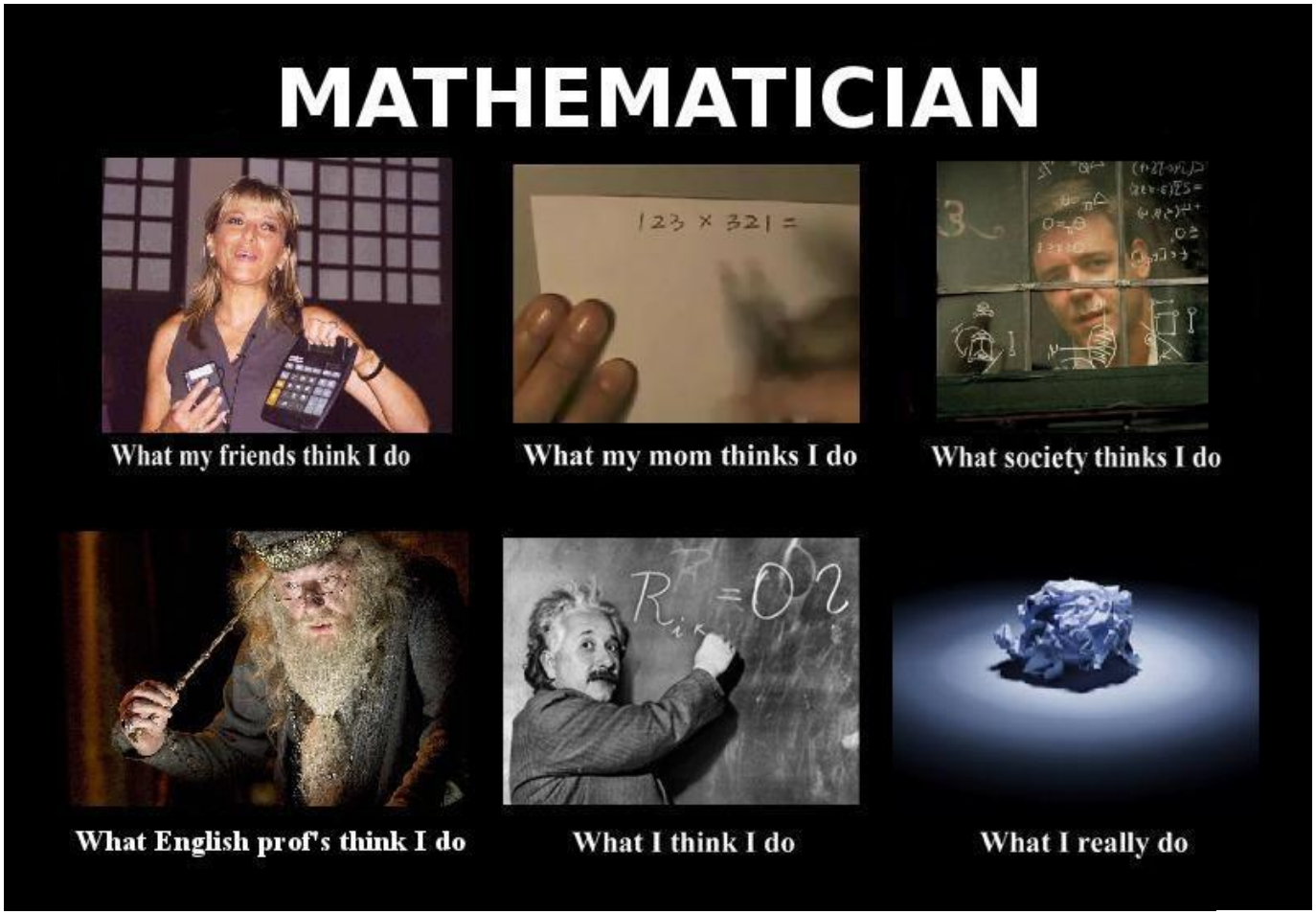
Une promenade (aléatoire ?) du côté de la recherche en mathématiques

Arnaud LIONNET

Mathématicien



INVENTEURS DU MONDE NUMÉRIQUE

Il reste des choses à découvrir en maths ?

We have not succeeded in answering all our problems. The answers we have found only serve to raise a whole set of new questions. In some ways we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things.

L'exemple le plus simple

La conjecture de Collatz / suite de Syracuse (~1920)

Prenez un nombre entier.

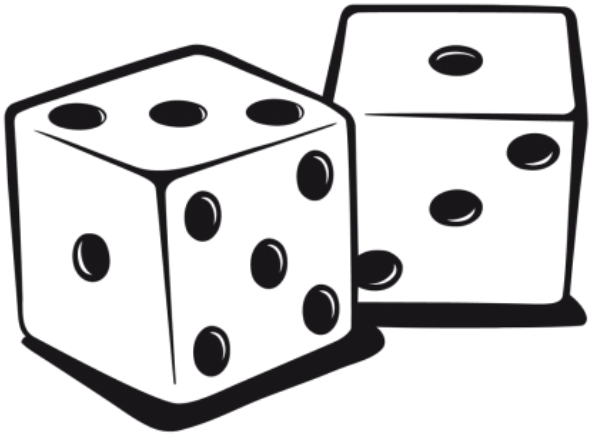
A chaque étape : si votre nombre est pair, divisez par 2.
S'il est impair, multipliez par 3 et ajoutez 1.

Exemple : 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, …

Essayez en partant de 7, ou 15.

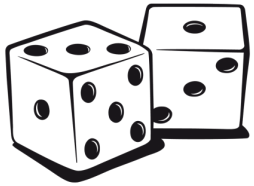
Les probabilités

L'étude (mathématique) du hasard.



Un sujet mathématique depuis fin 16e début 17e siècle.

Les probabilités aujourd'hui



Google

Quelques domaines en probabilités

Environnement aléatoires.

Files d'attente.

Modèles de population.

Les marches aléatoires.

Les méthodes numériques probabilistes.

Ce qu’on fait en probabilités

Calculer la probabilité d’un événement.

Décrire ce qu’il se passe, dire des choses, ce qui peut ou ne peut pas se passer.

Y a t’il des choses qu’on peut prédire malgré tout ?

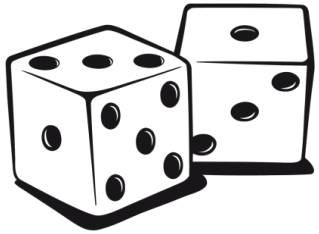
Vous me demandez de vous prédire les phénomènes qui vont se produire. Si, par malheur, je connaissais les lois de ces phénomènes, je ne pourrais y arriver que par des calculs inextricables et je devrais renoncer à vous répondre ; mais, comme j’ai la chance de les ignorer, je vais vous répondre tout de suite. Et, ce qu’il y a de plus extraordinaire, c’est que ma réponse sera juste.

Henri Poincaré.

La loi des grand nombres

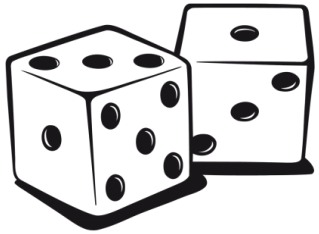
On tire un dé.

P(obtenir 1)=P(obtenir …)=1/6.

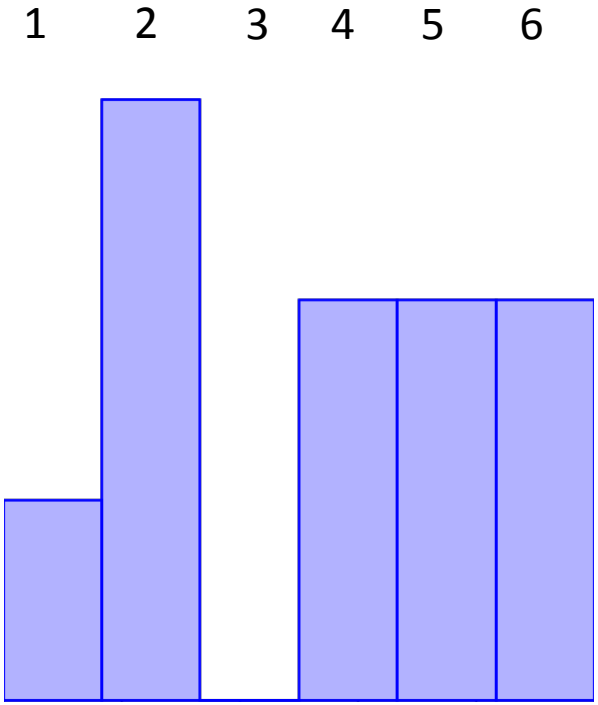


La loi des grand nombres

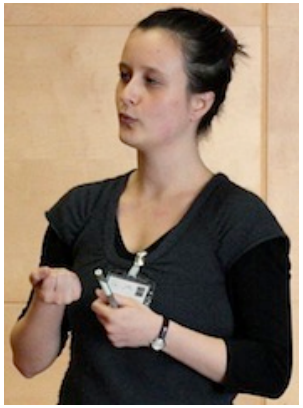
On tire un dé.
P(obtenir 1)=P(obtenir …)=1/6.



10 tirages.

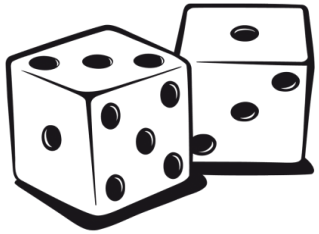


(aimablement prêté par Cécile Mailler)

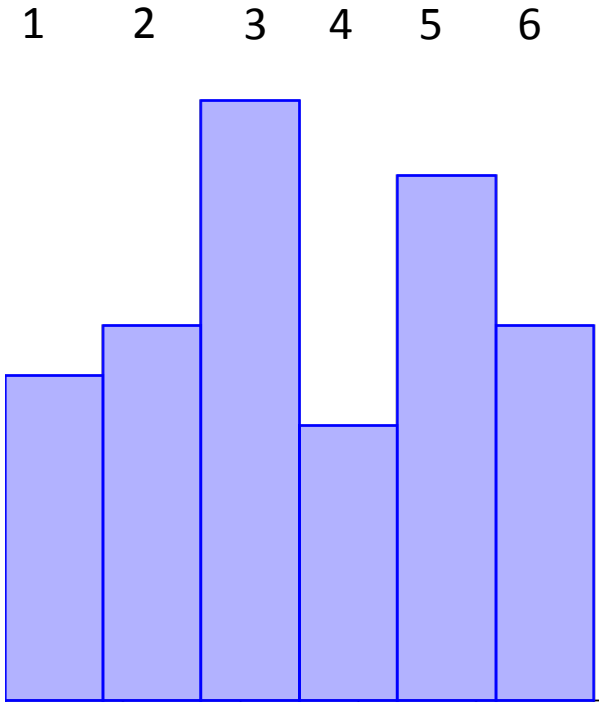


La loi des grand nombres

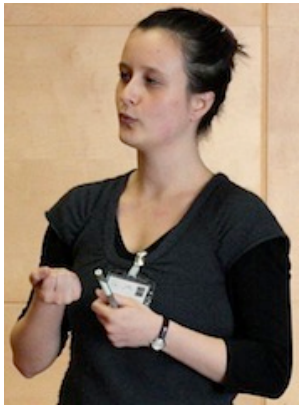
On tire un dé.
P(obtenir 1)=P(obtenir …)=1/6.



100 tirages.

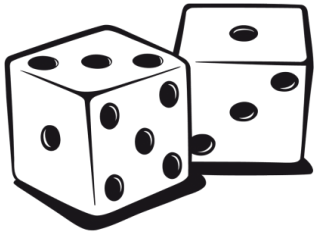


(aimablement prêté par Cécile Mailler)

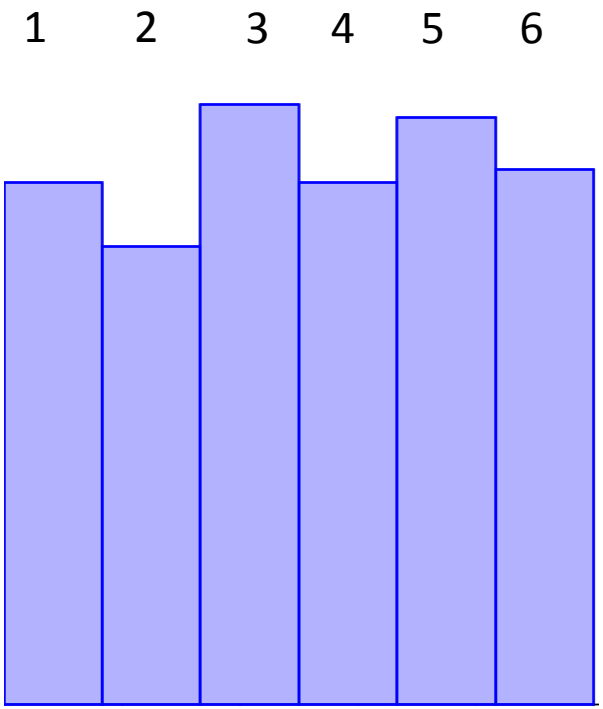


La loi des grand nombres

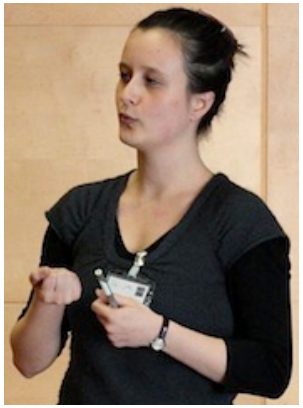
On tire un dé.
P(obtenir 1)=P(obtenir …)=1/6.



500 tirages.



(aimablement prêté par Cécile Mailler)



La loi des grand nombres

Loi de Murphy ?

Il y a souvent des gagnants au loto.

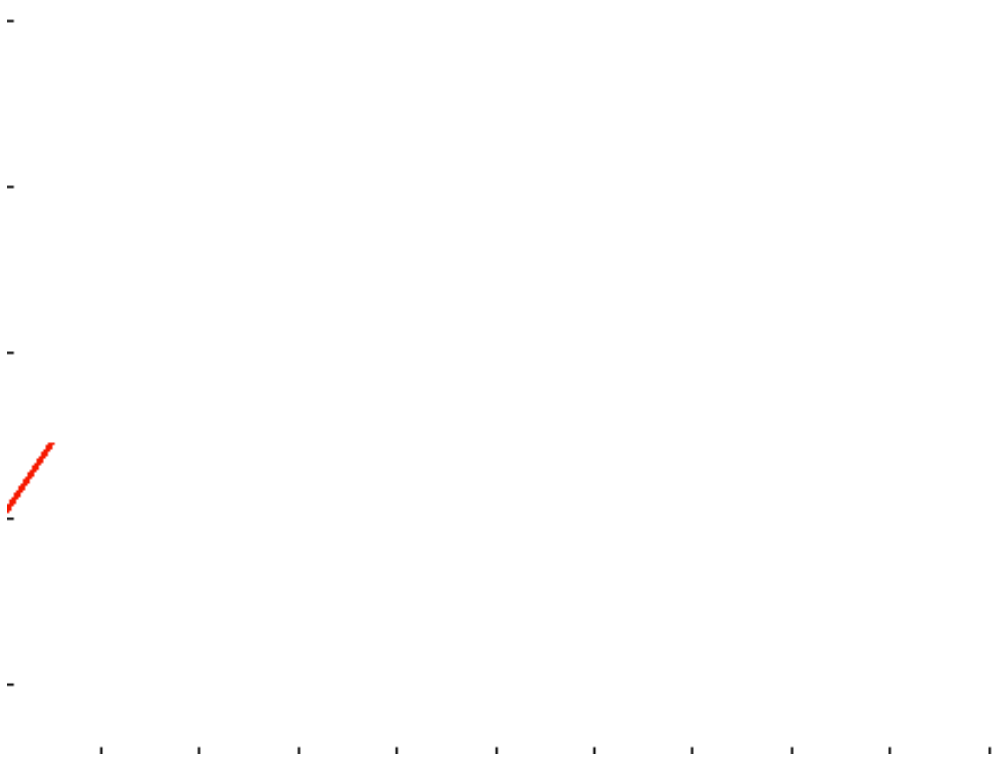
Trop improbable, ça ne pourra pas arriver ?

Il suffit d'essayer suffisamment de fois !

Marche aléatoire

On tire à pile ou face.

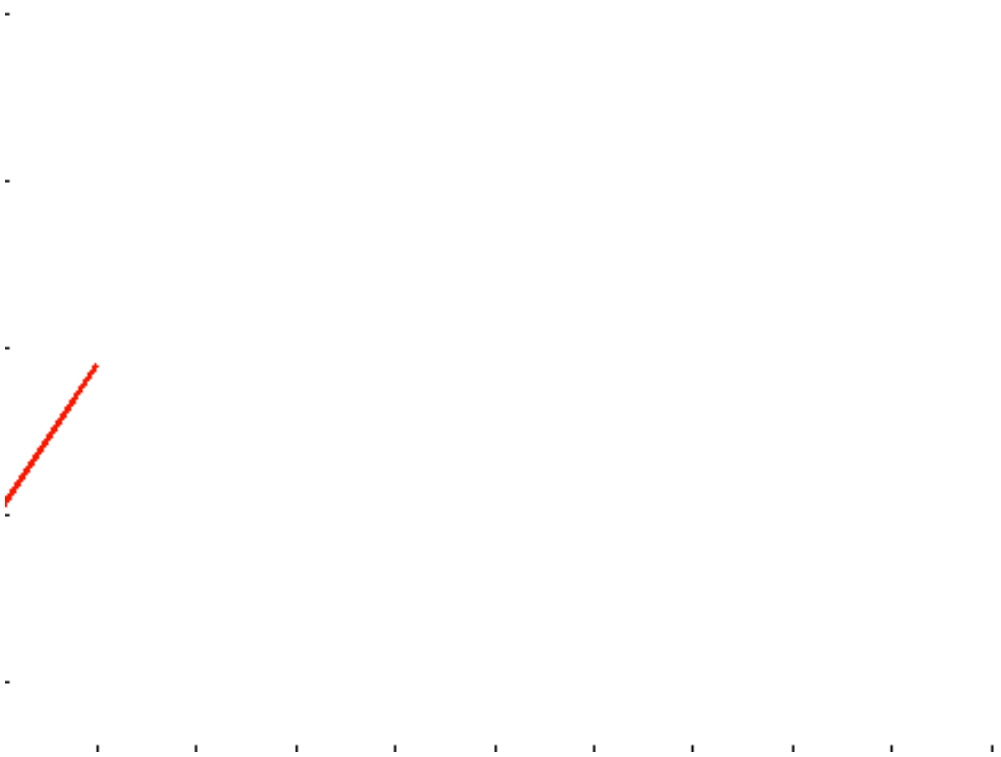
Pile on monte de 1, face on descend de 1.



Marche aléatoire

On tire à pile ou face.

Pile on monte de 1, face on descend de 1.



Marche aléatoire

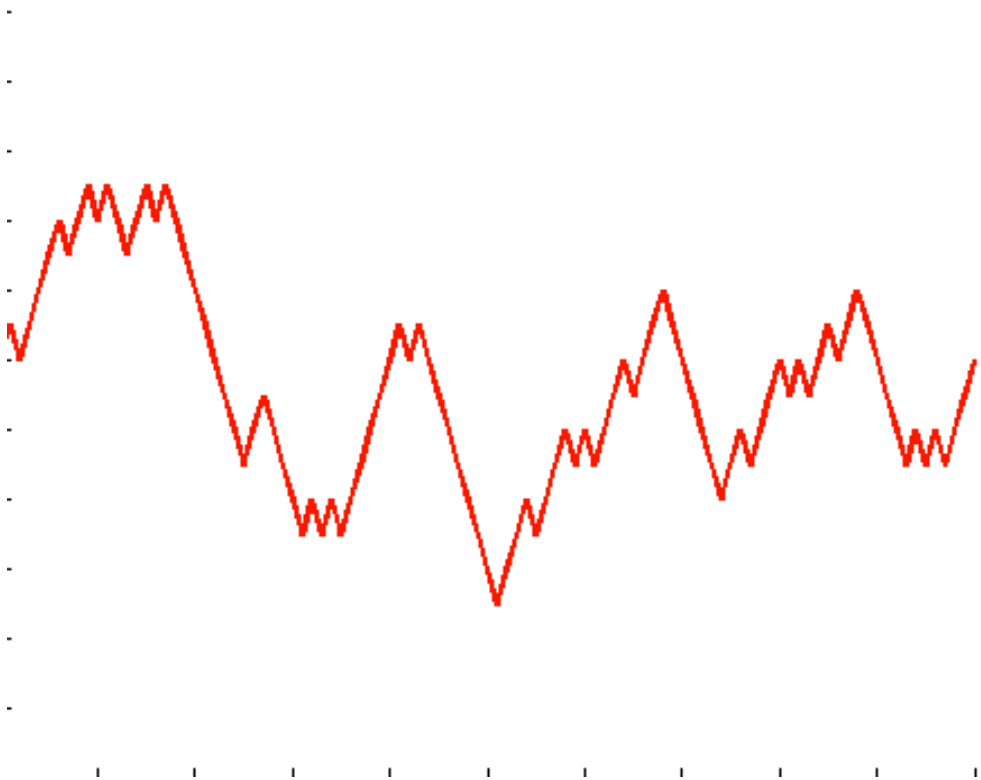
On tire à pile ou face.

Pile on monte de 1, face on descend de 1.

Marche aléatoire

On tire à pile ou face.

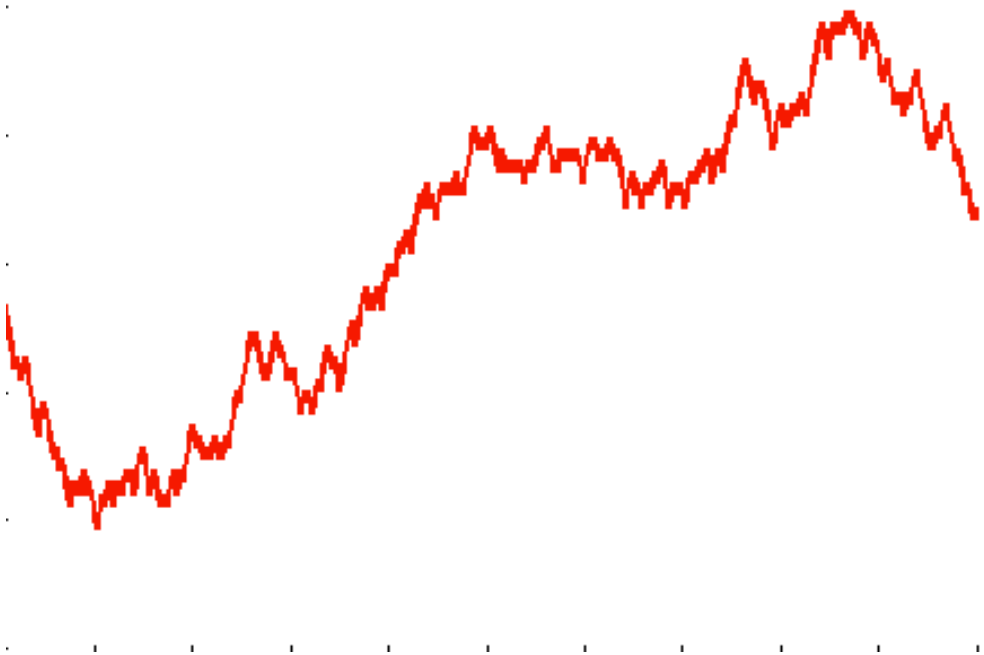
Pile on monte de 1, face on descend de 1.



Marche aléatoire

On tire à pile ou face.

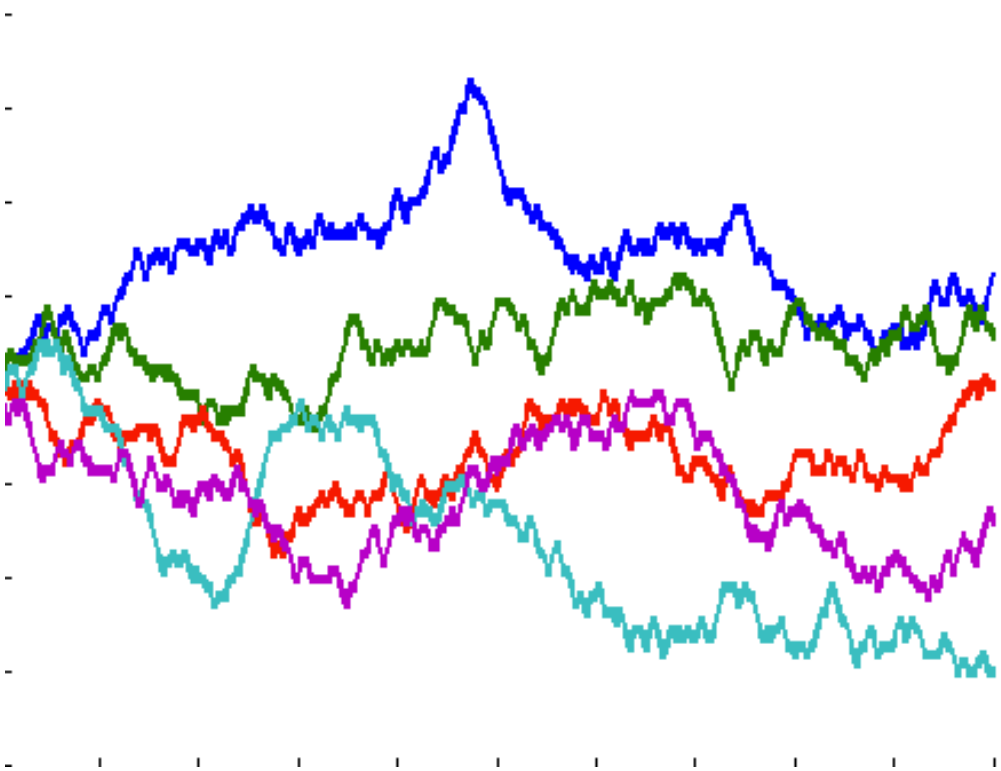
Pile on monte de 1, face on descend de 1.



Marche aléatoire

On tire à pile ou face.

Pile on monte de 1, face on descend de 1.



Ce que je fais

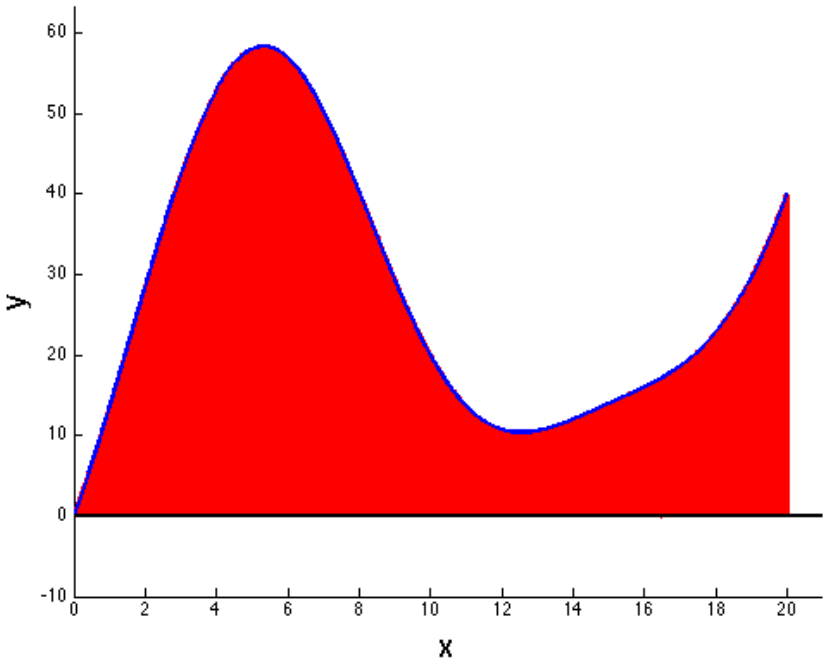
Je développe des méthodes numériques

probabilistes pour résoudre des équations.

(Des équations aux dérivées partielles.)

Intégrales

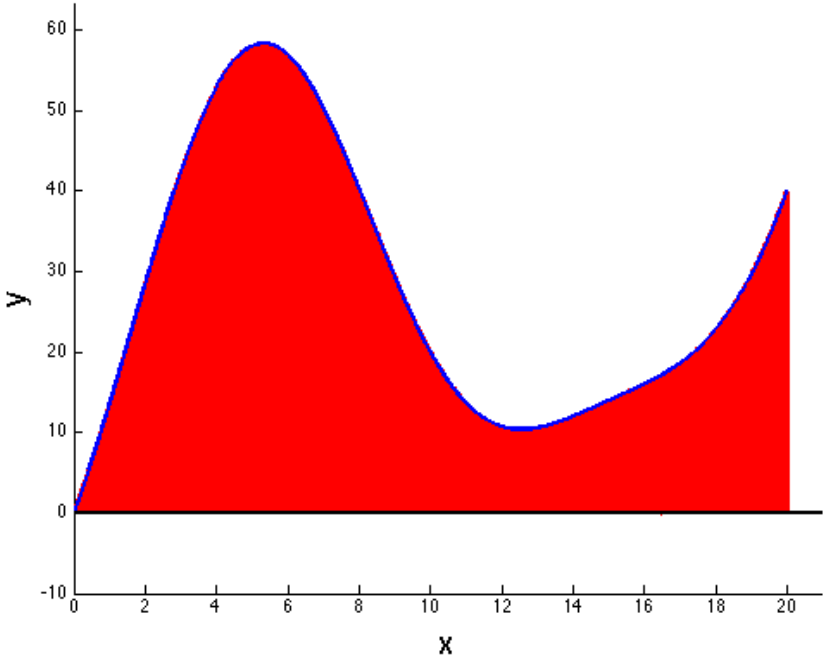
Calculer l’aire sous une courbe.



Intégrales

Calculer l'aire sous une courbe.

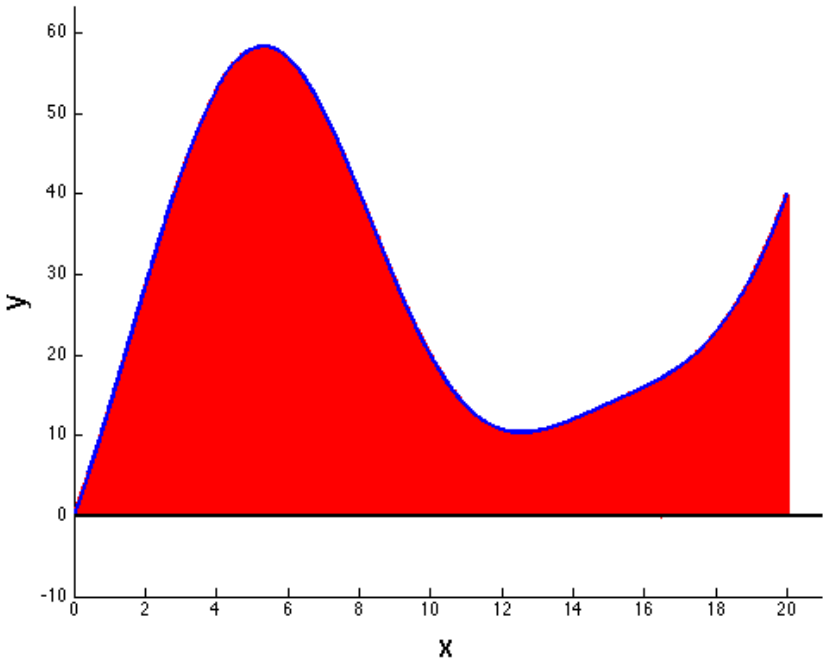
Motivation ?



Intégrales

Calculer l'aire sous une courbe.

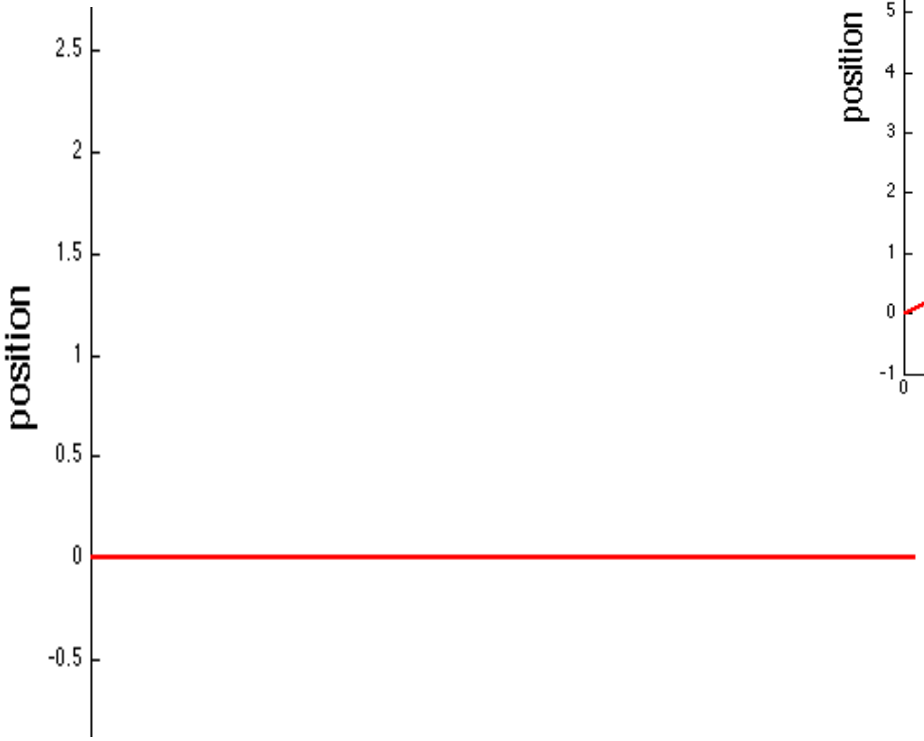
Motivation ?

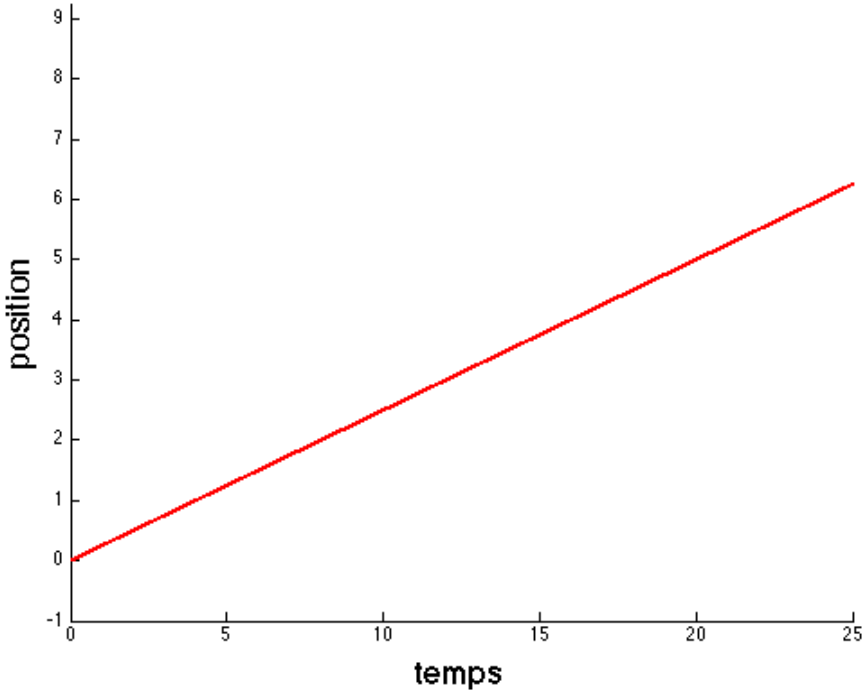


Loi de Newton :

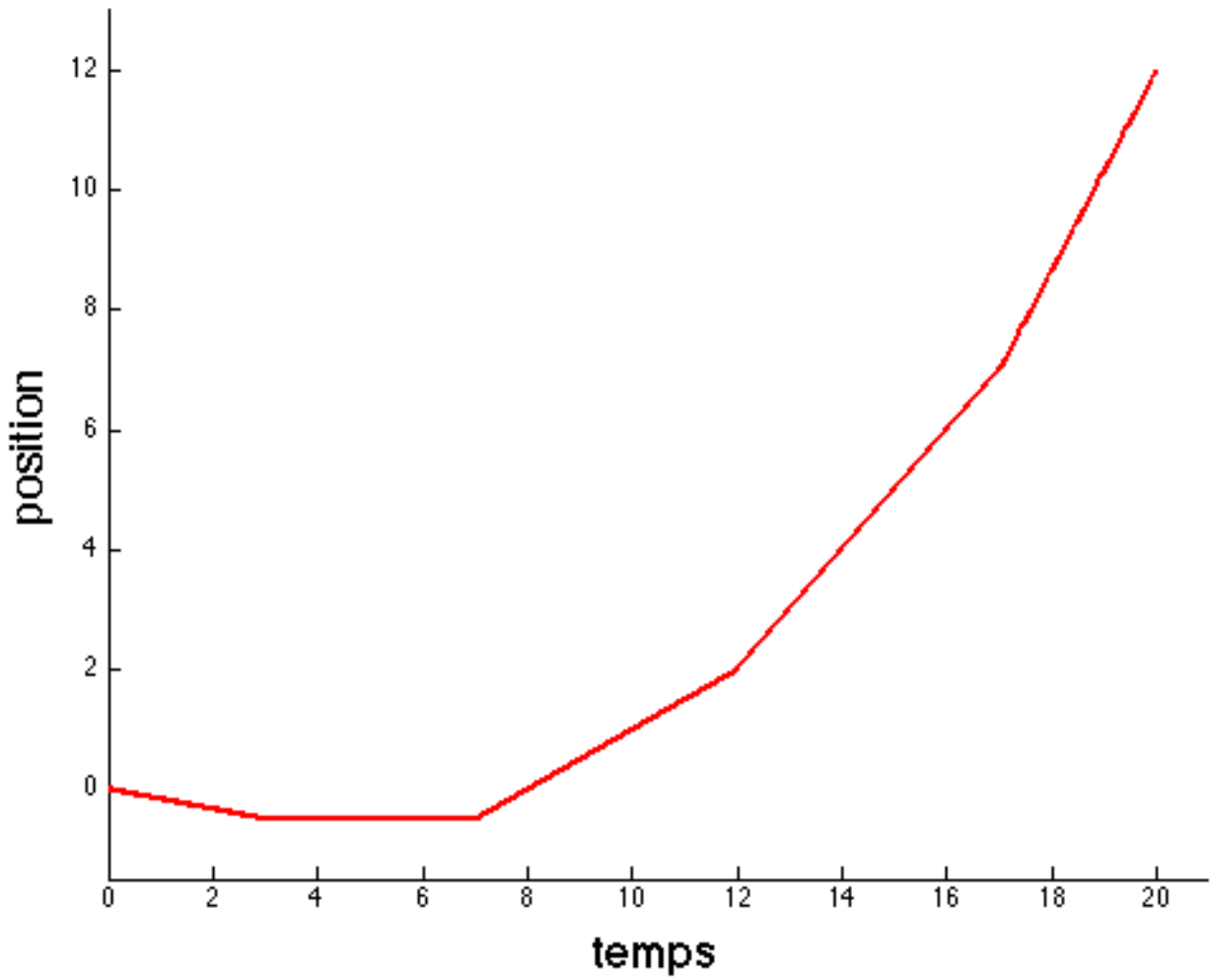
masse×accélération = somme des forces.

position -> vitesse -> accélération

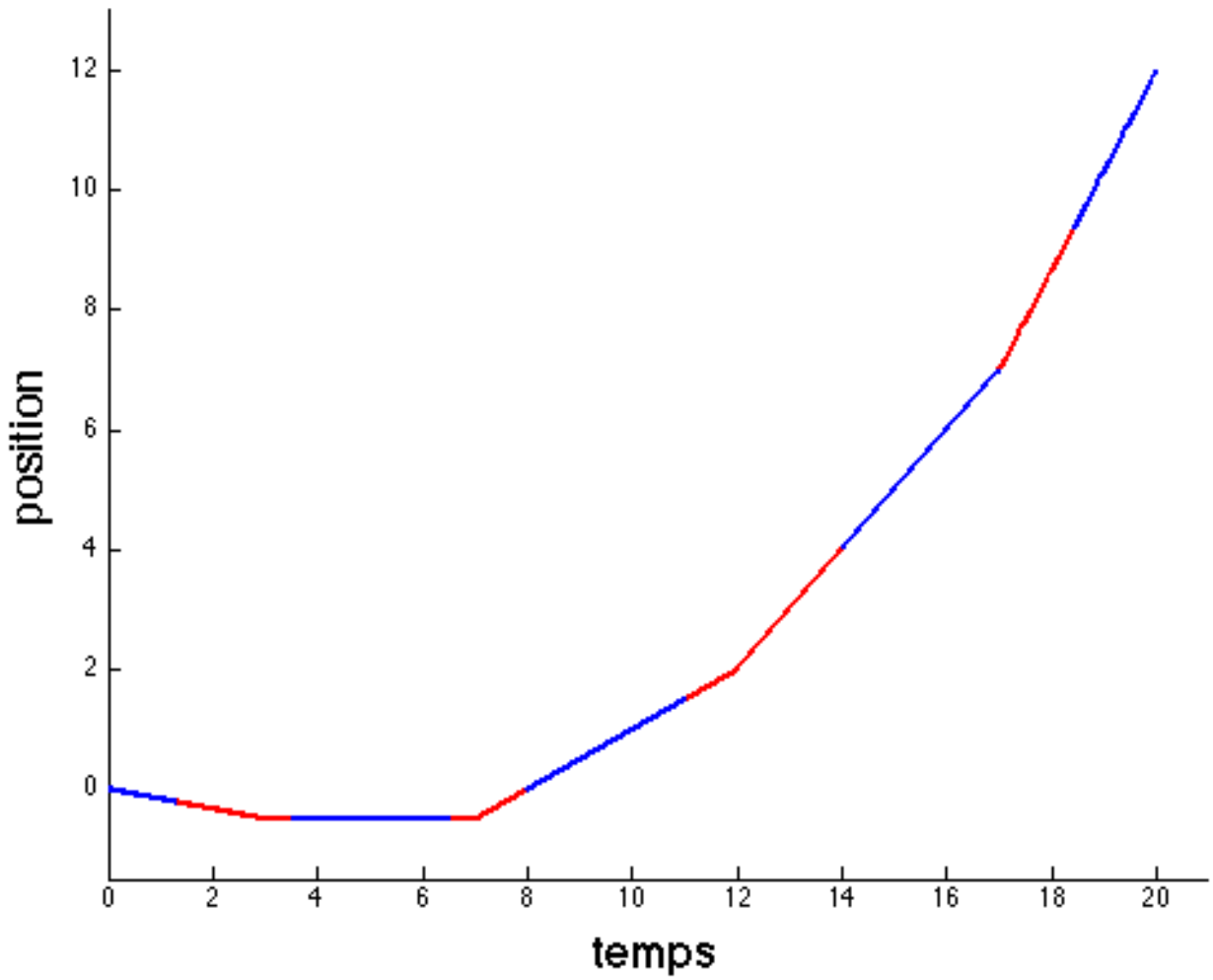




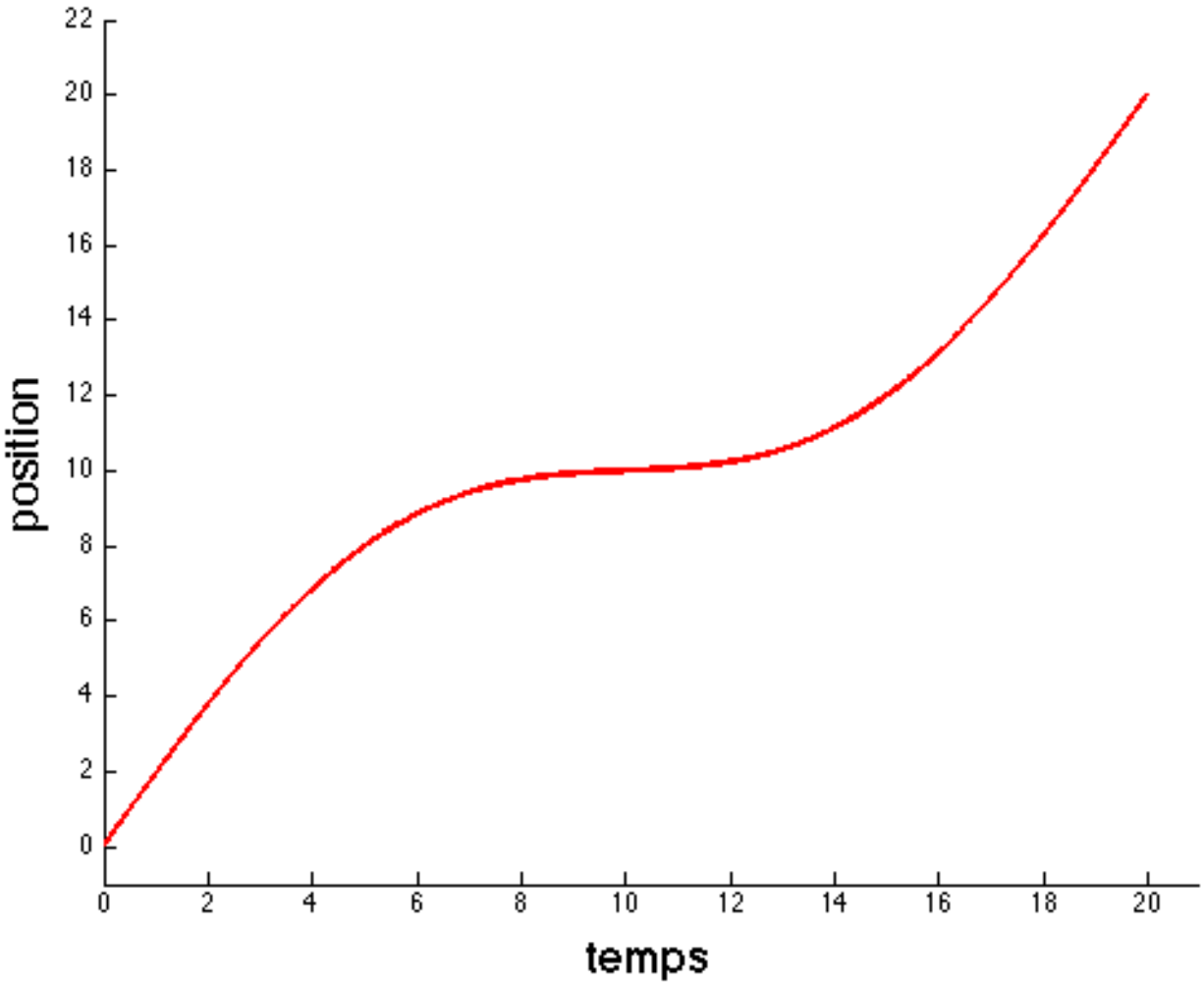
position -> vitesse -> accélération



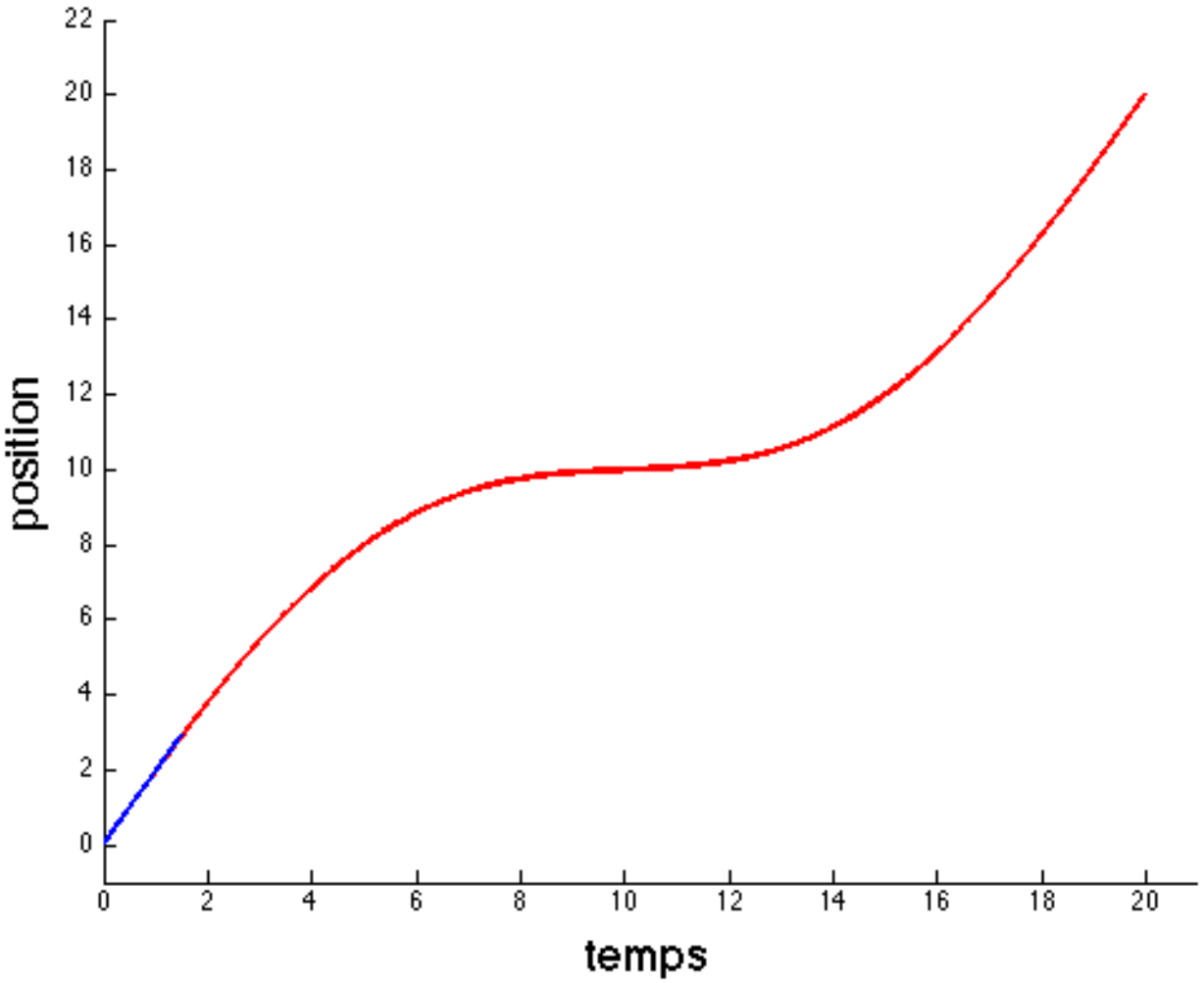
position -> vitesse -> accélération



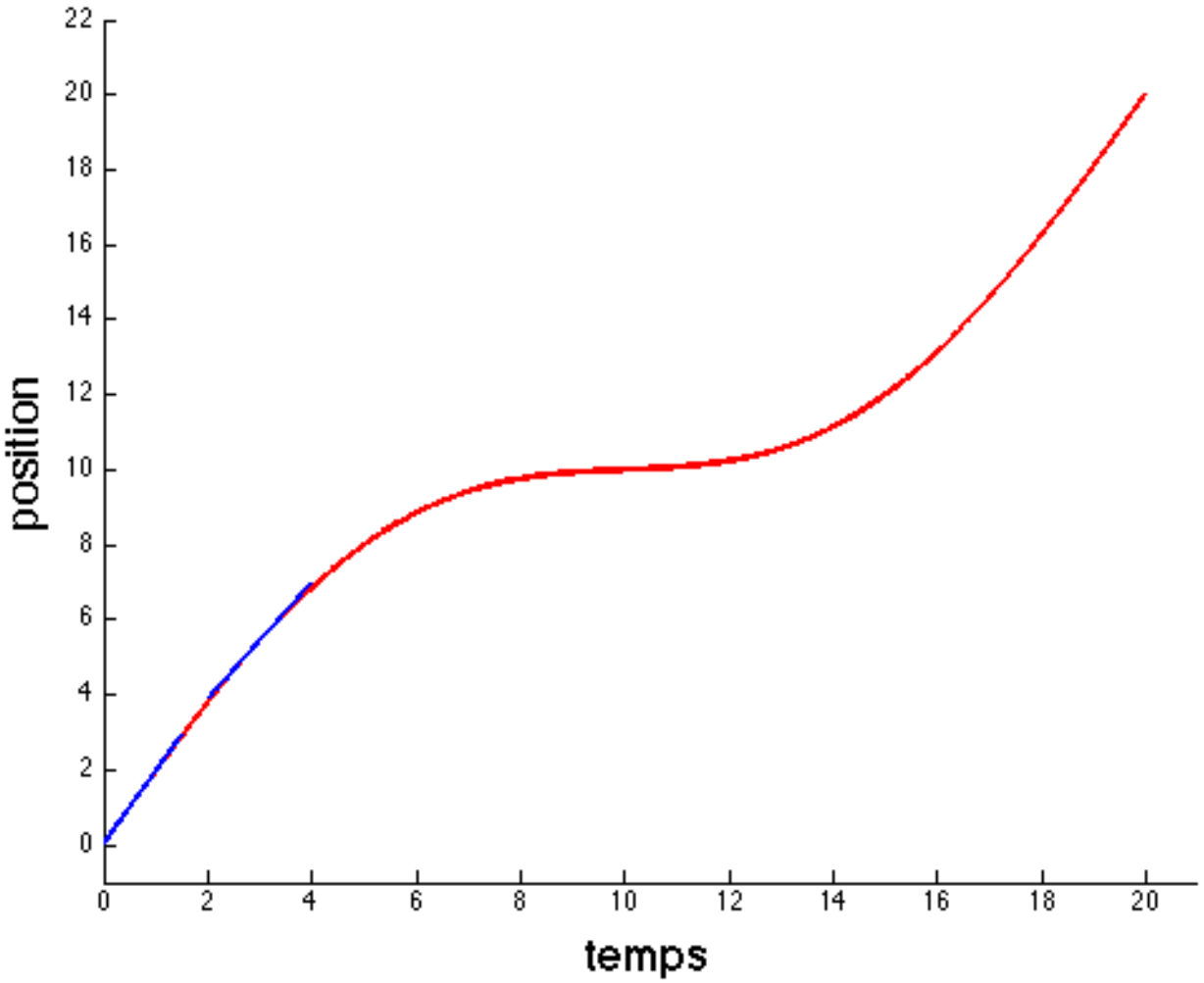
position -> vitesse -> accélération



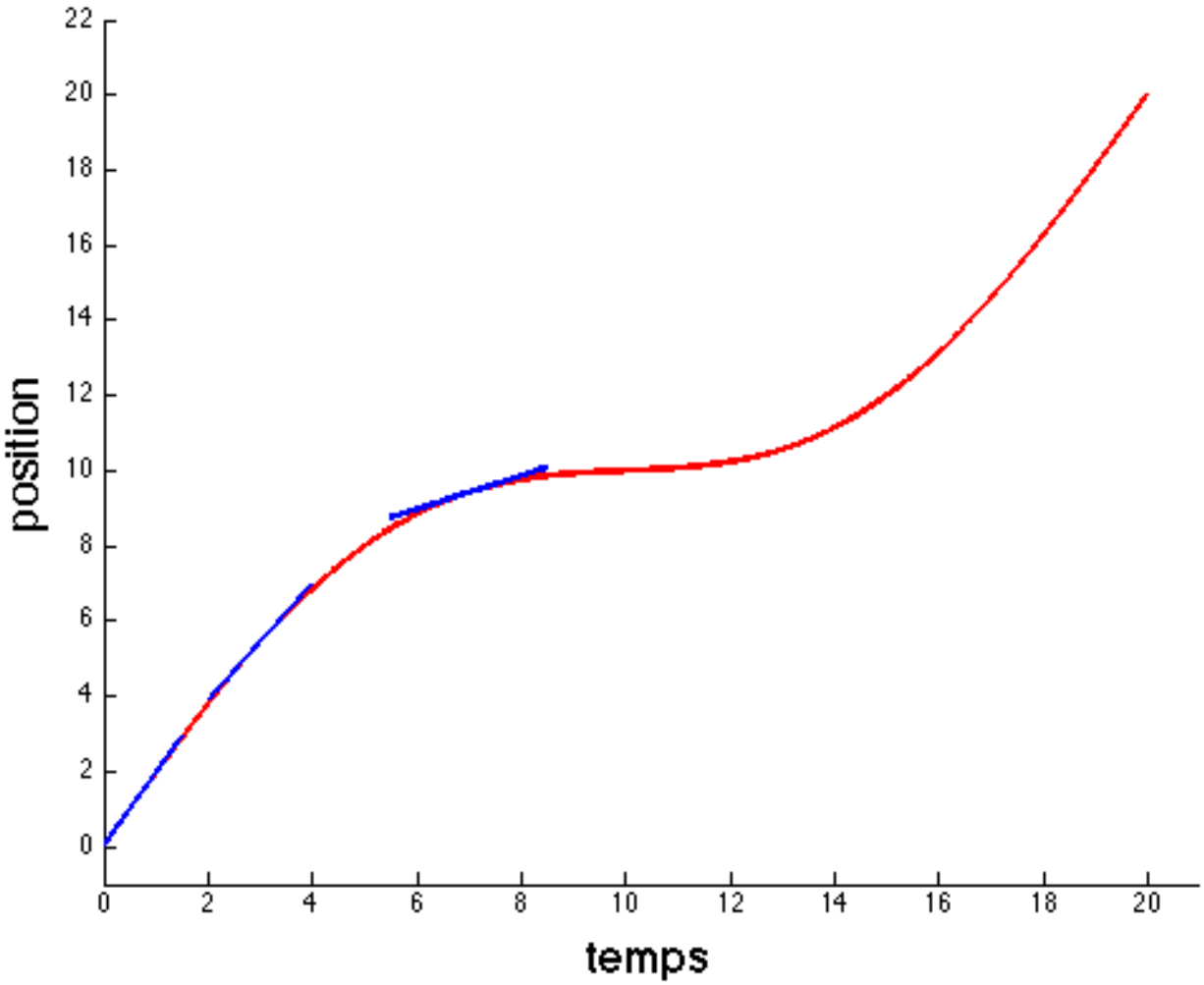
position -> vitesse -> accélération



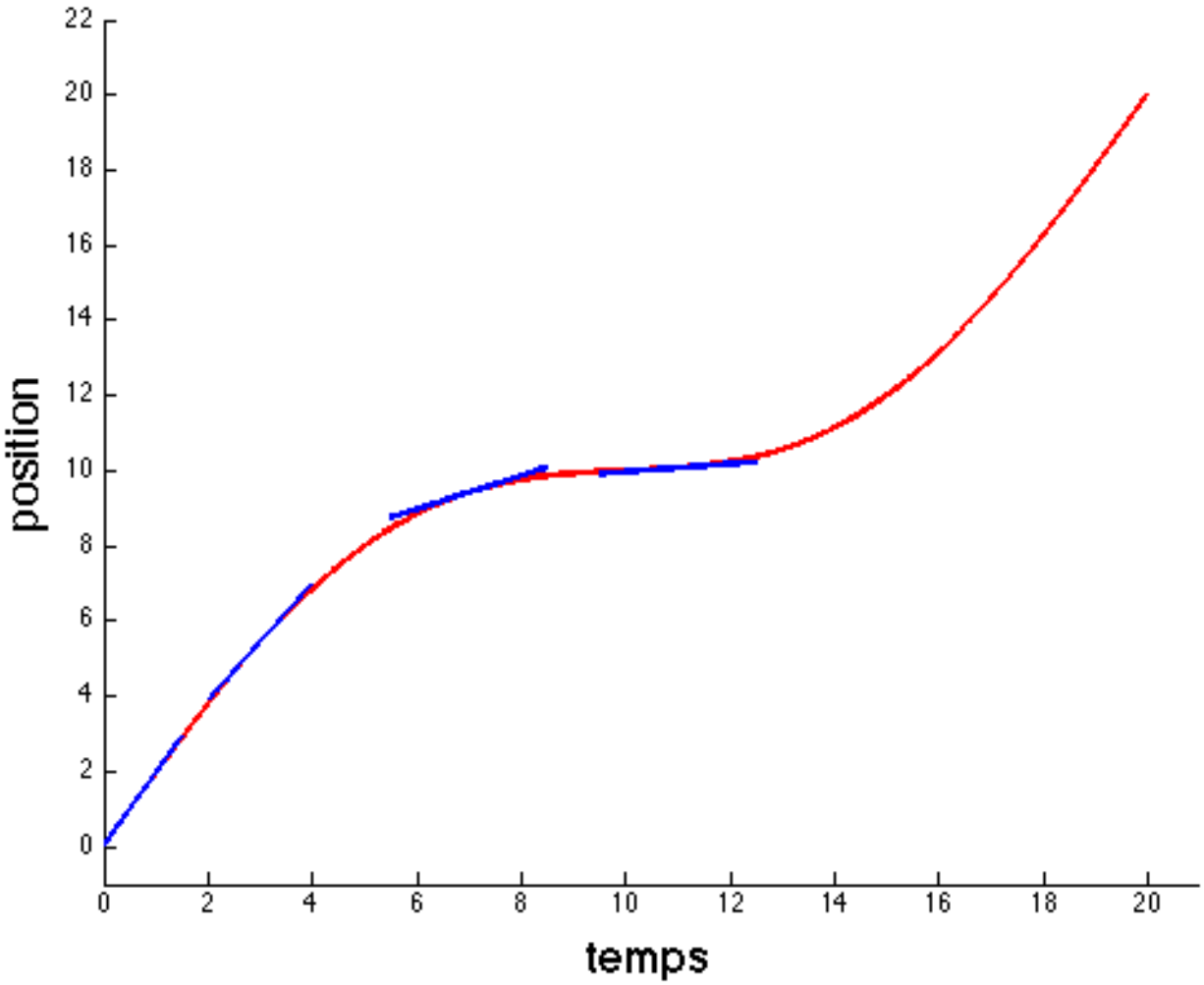
position -> vitesse -> accélération



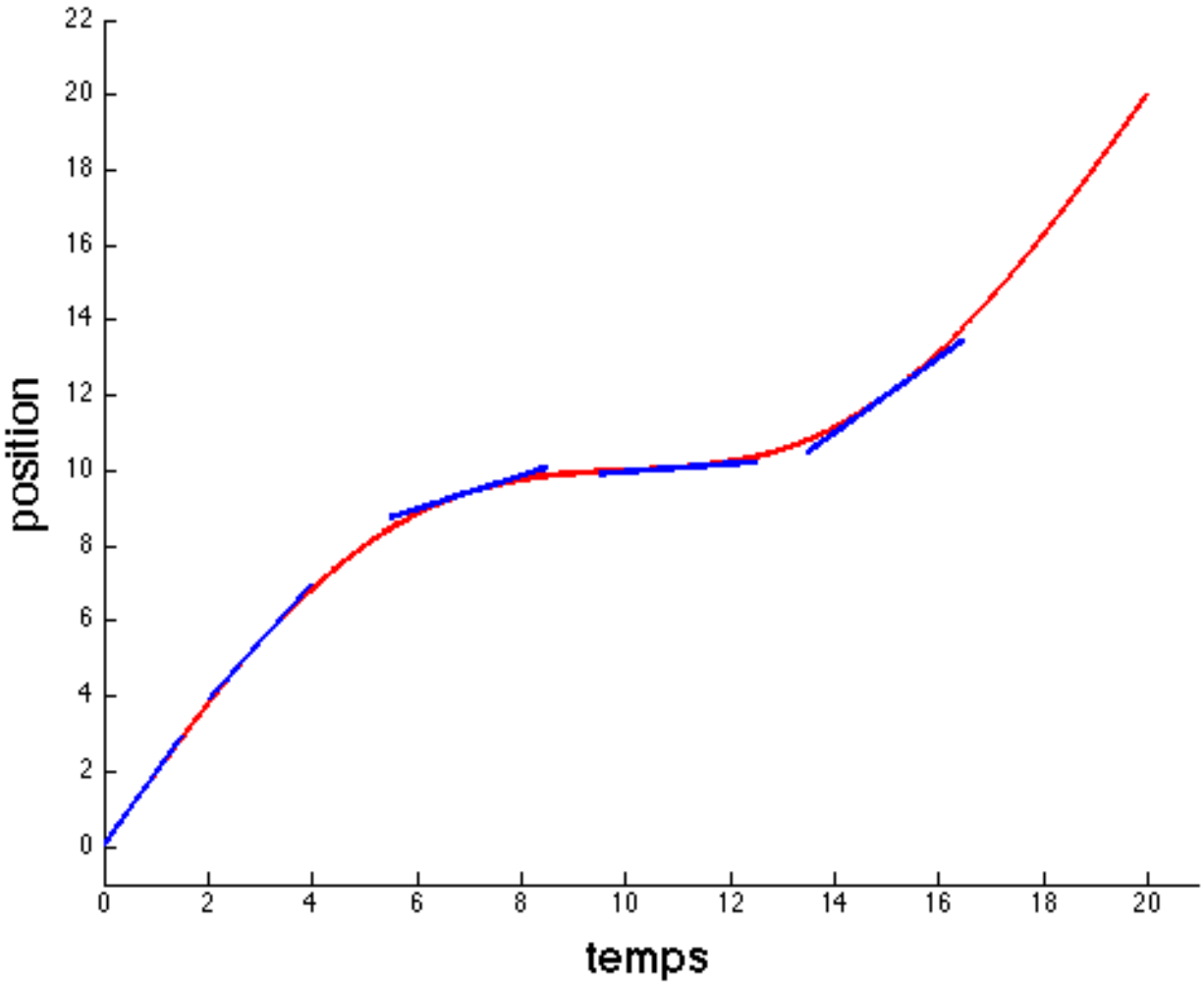
position -> vitesse -> accélération



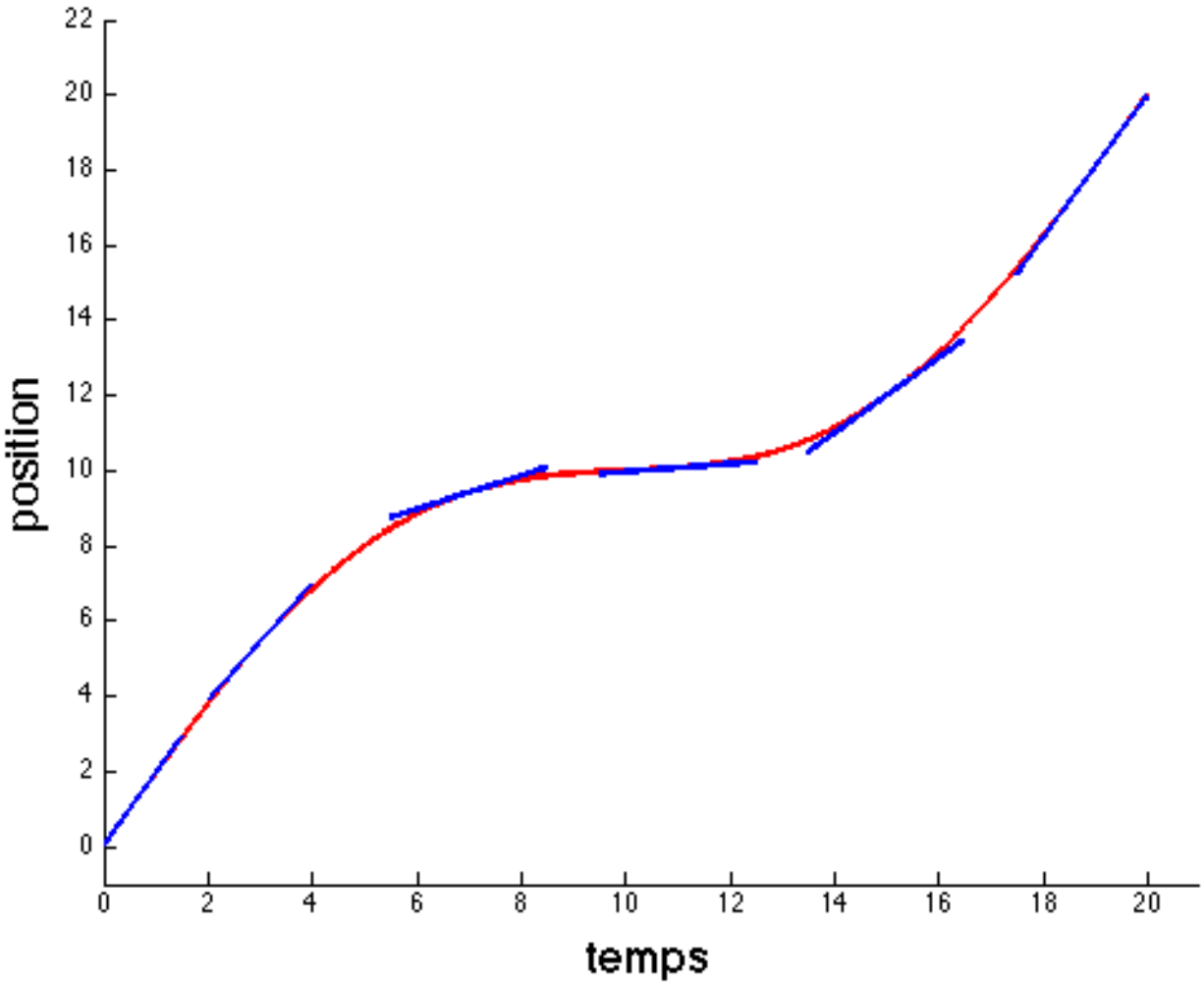
position -> vitesse -> accélération



position -> vitesse -> accélération



position -> vitesse -> accélération



Calcul différentiel (fin 17e siècle).

Newton

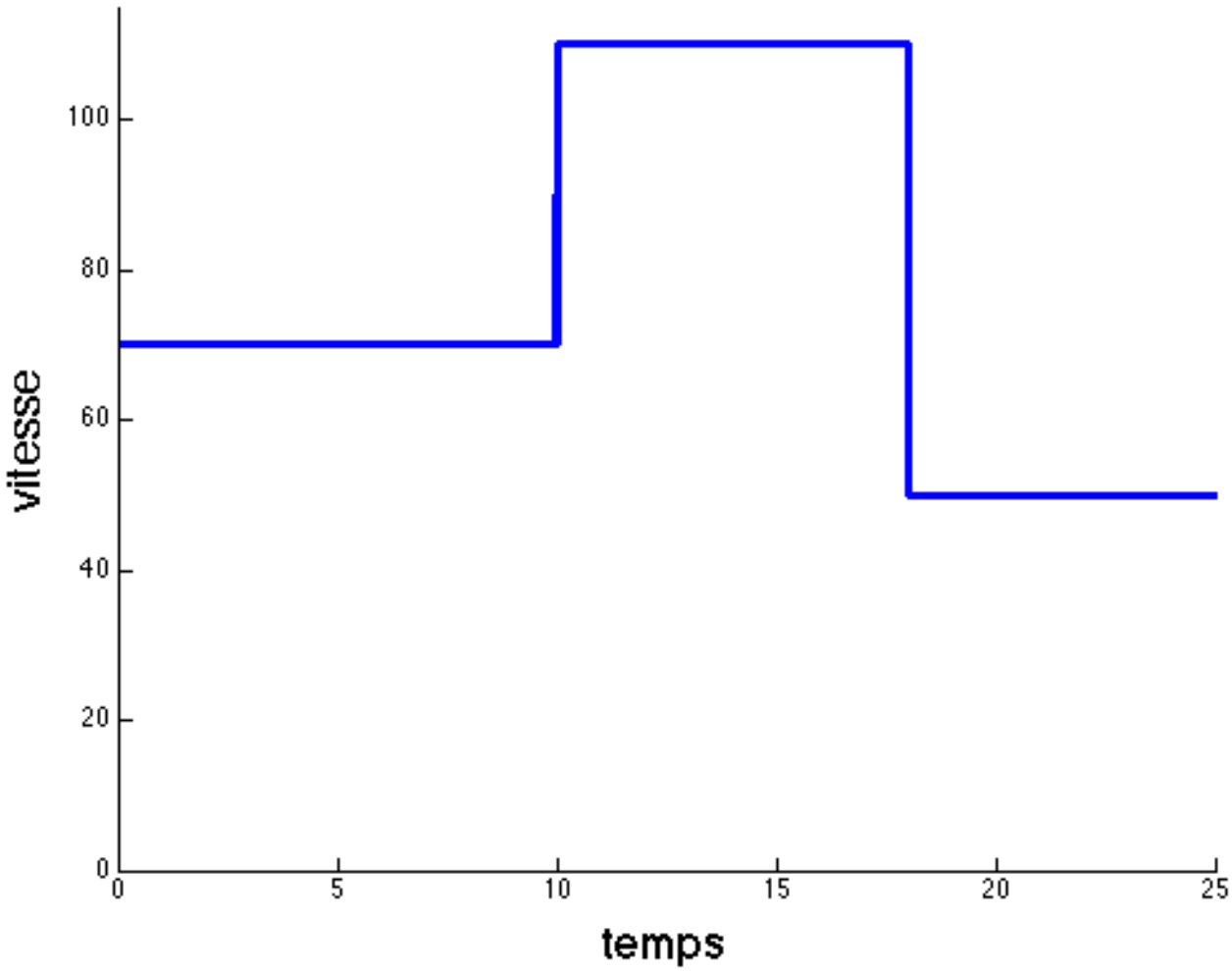
Leibniz

Règles de calcul.

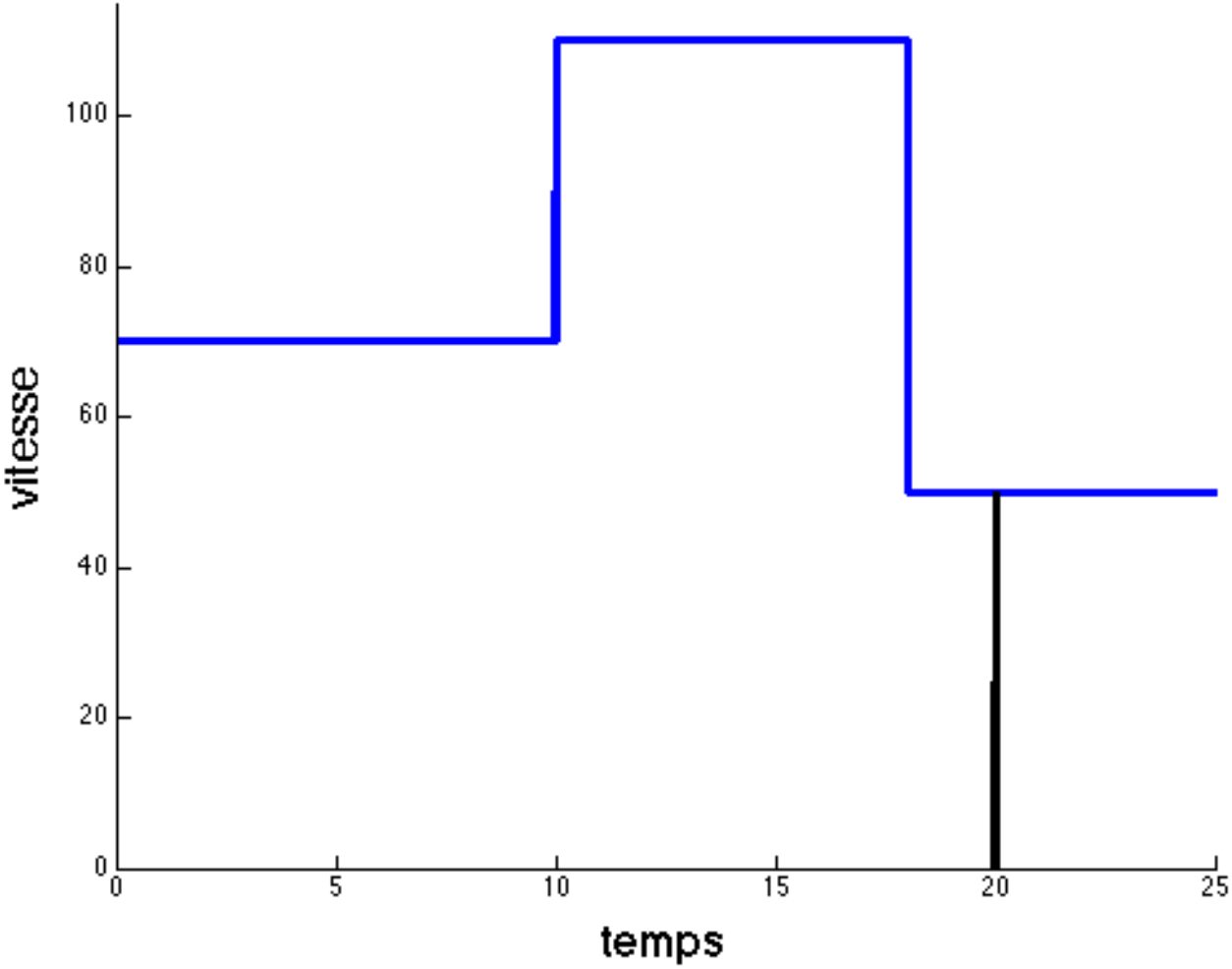
position(t) -> vitesse(t) -> accélération(t)

… Et dans l'autre sens ?

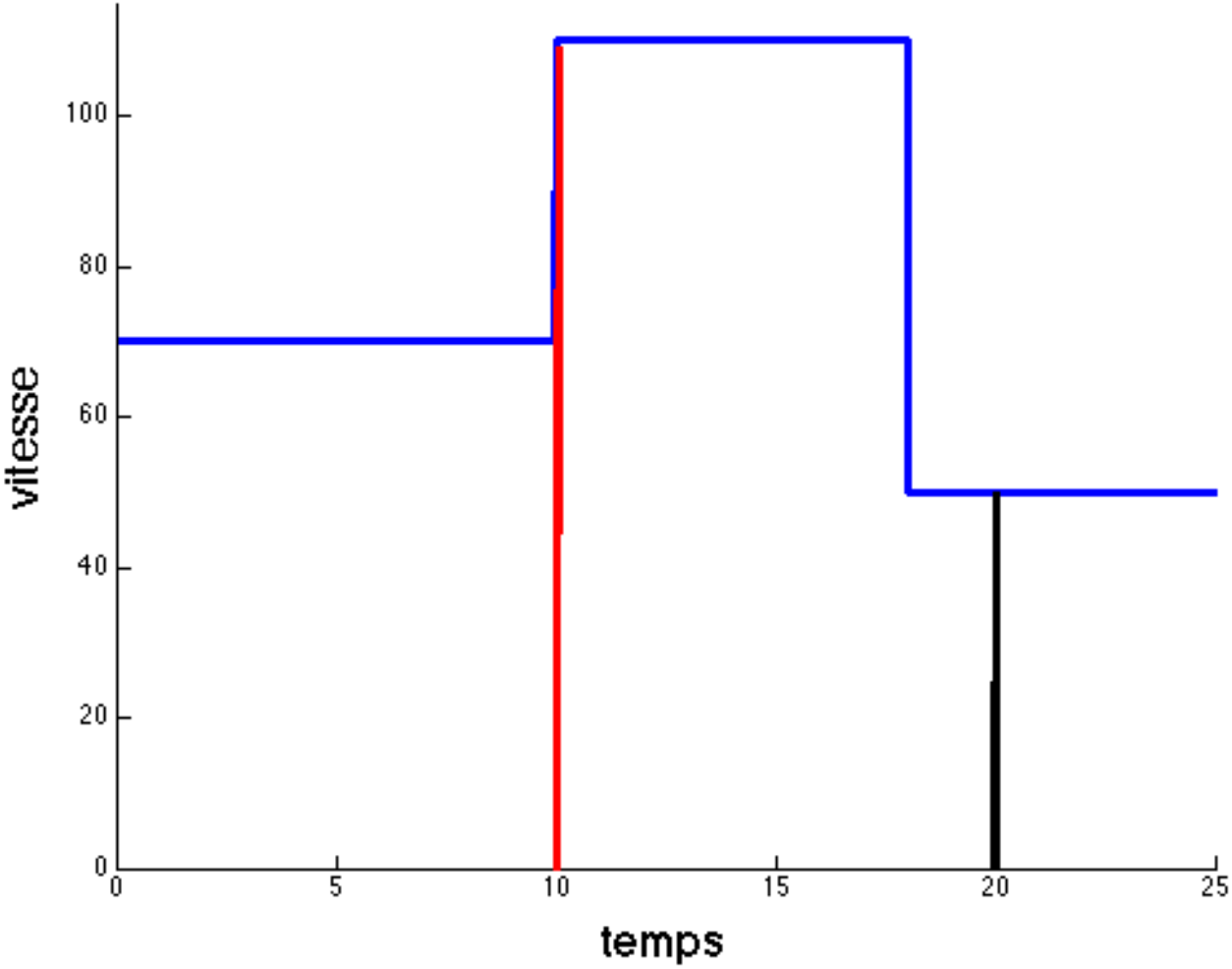
vitesse -> position



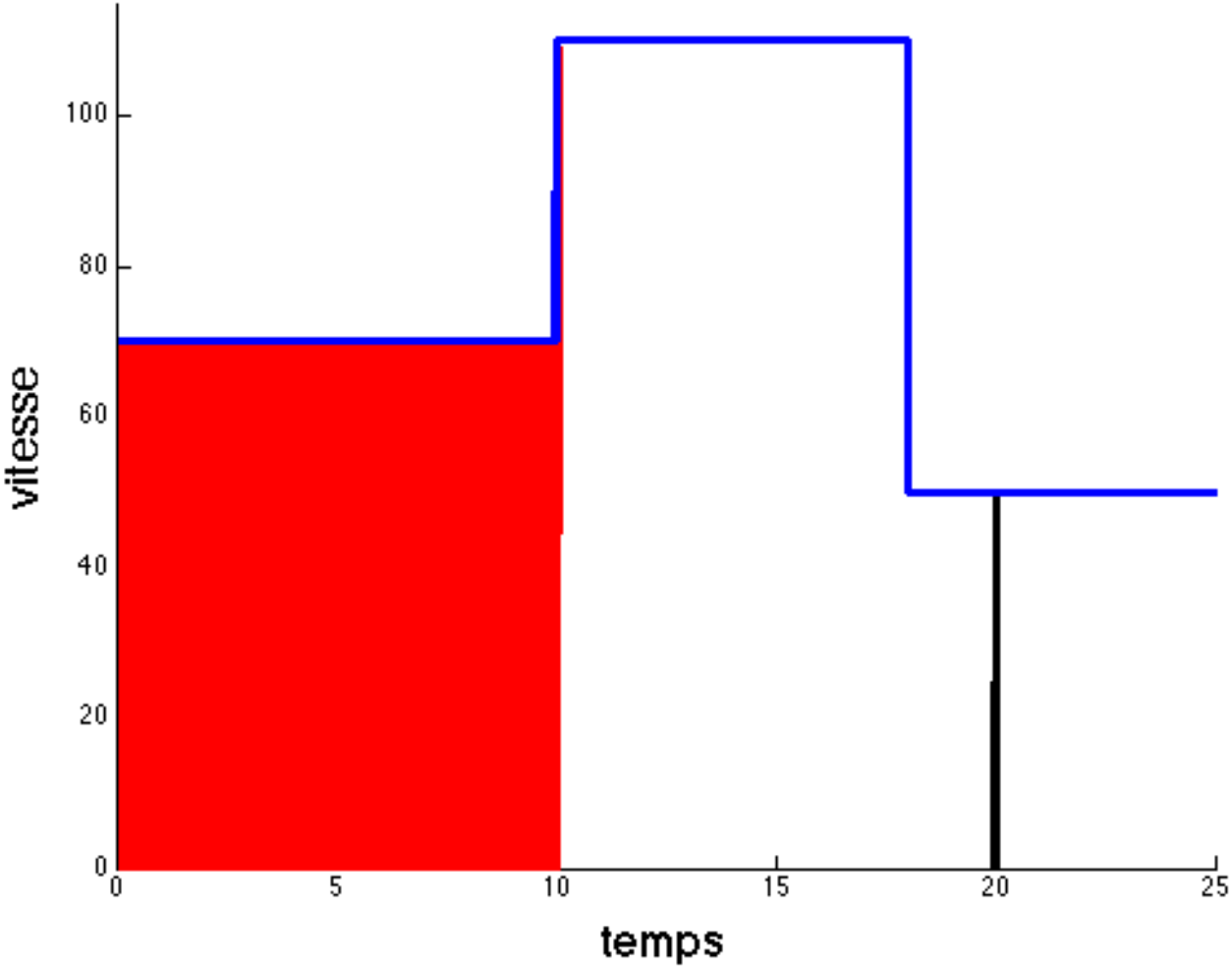
vitesse -> position



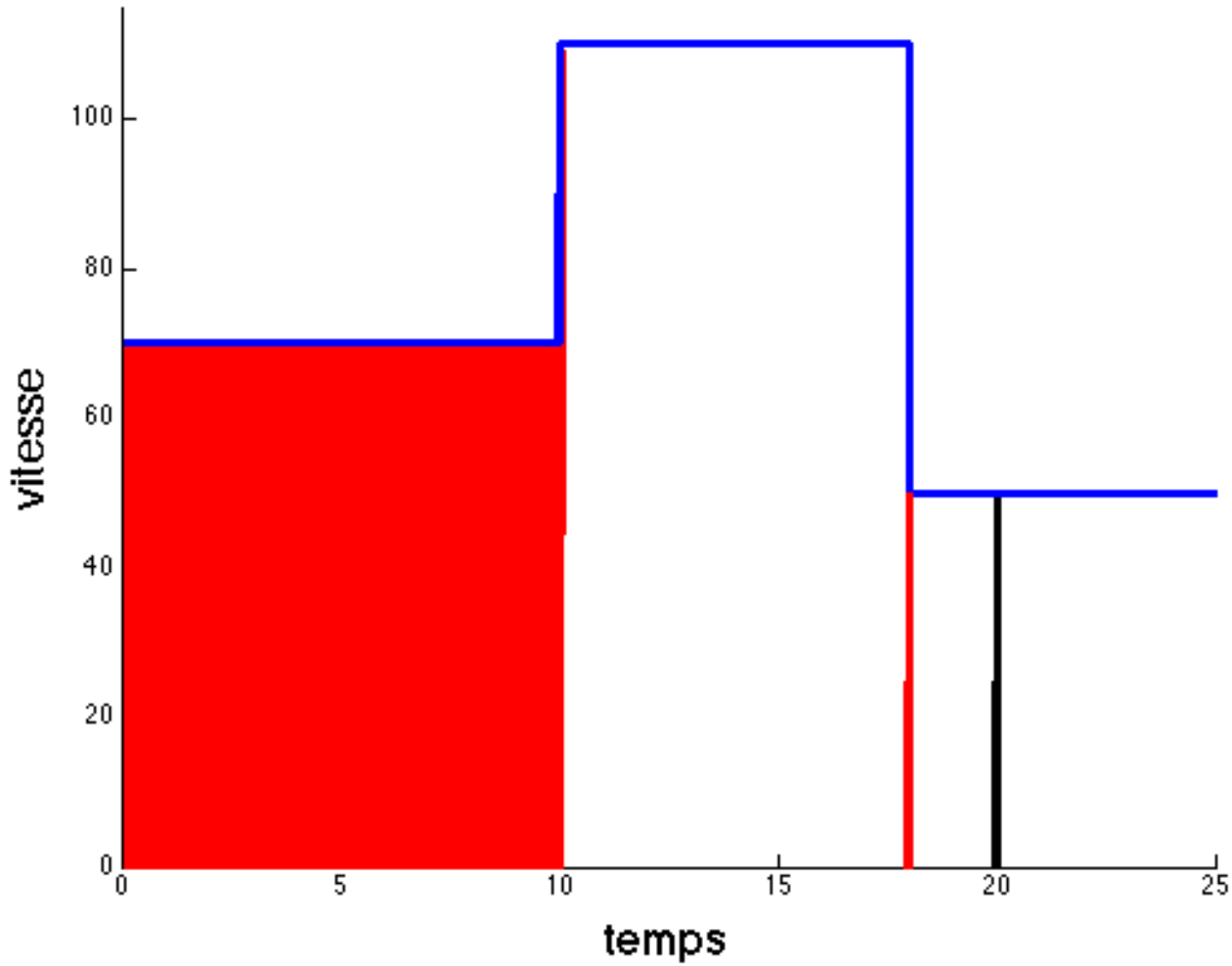
vitesse -> position



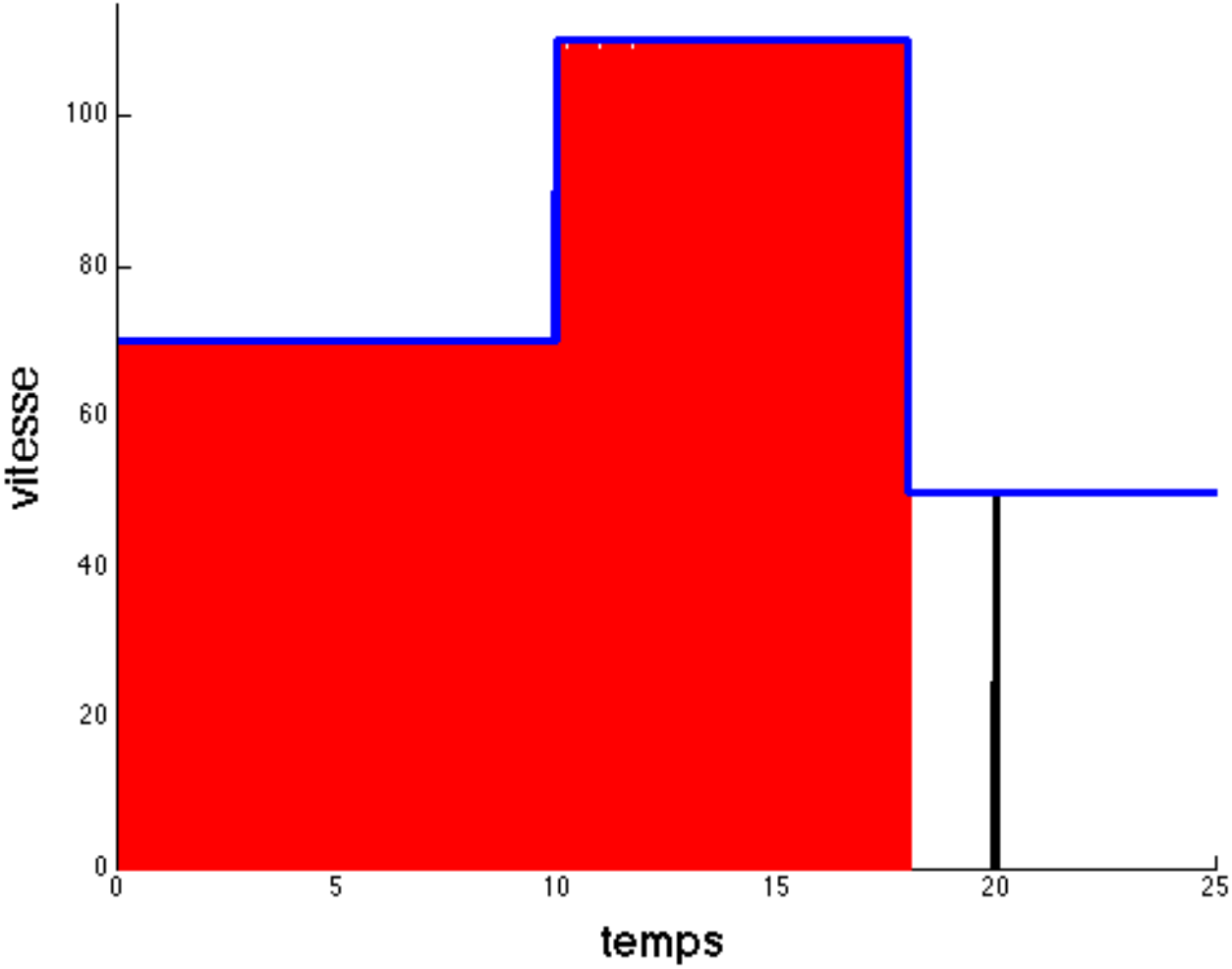
vitesse -> position



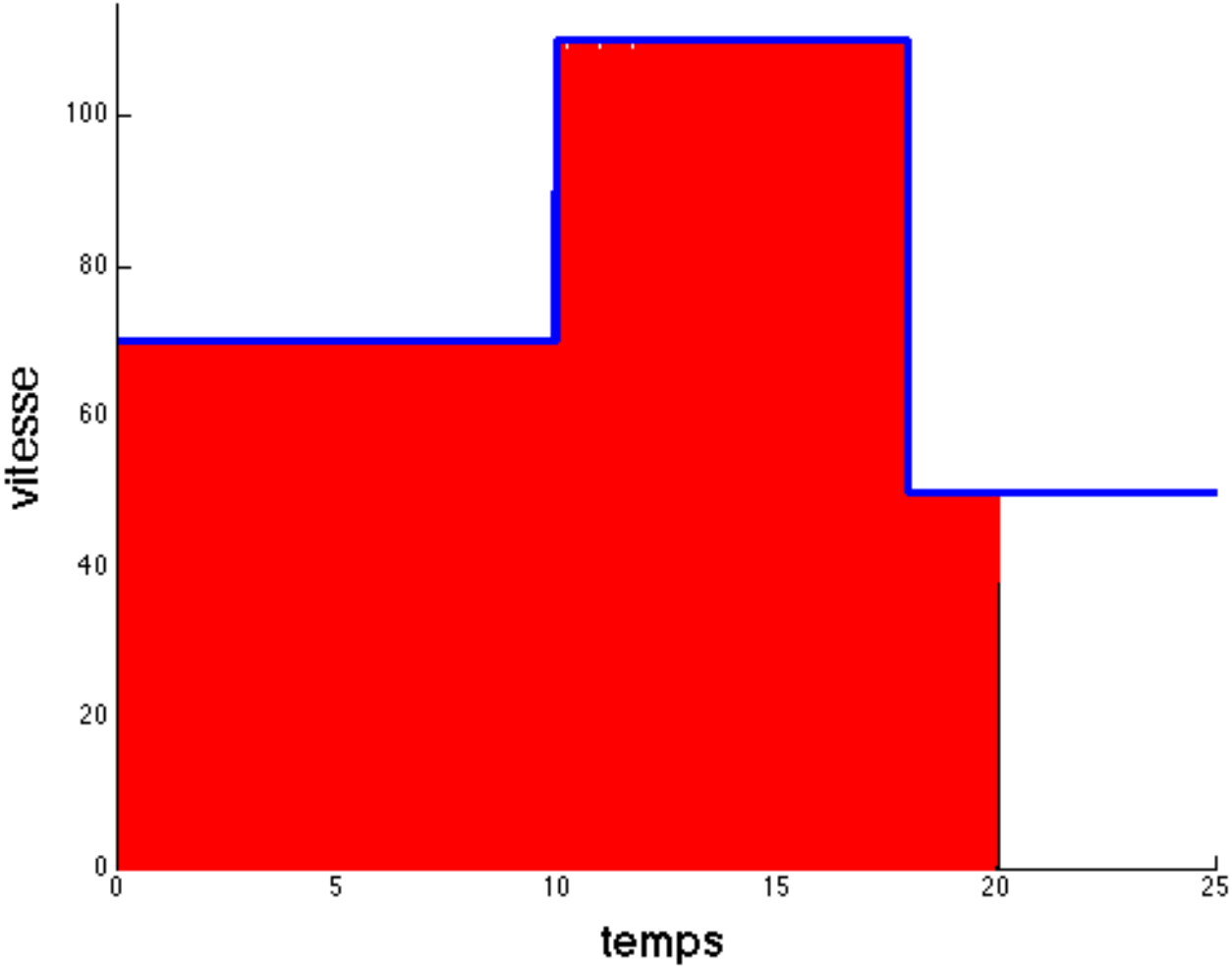
vitesse -> position



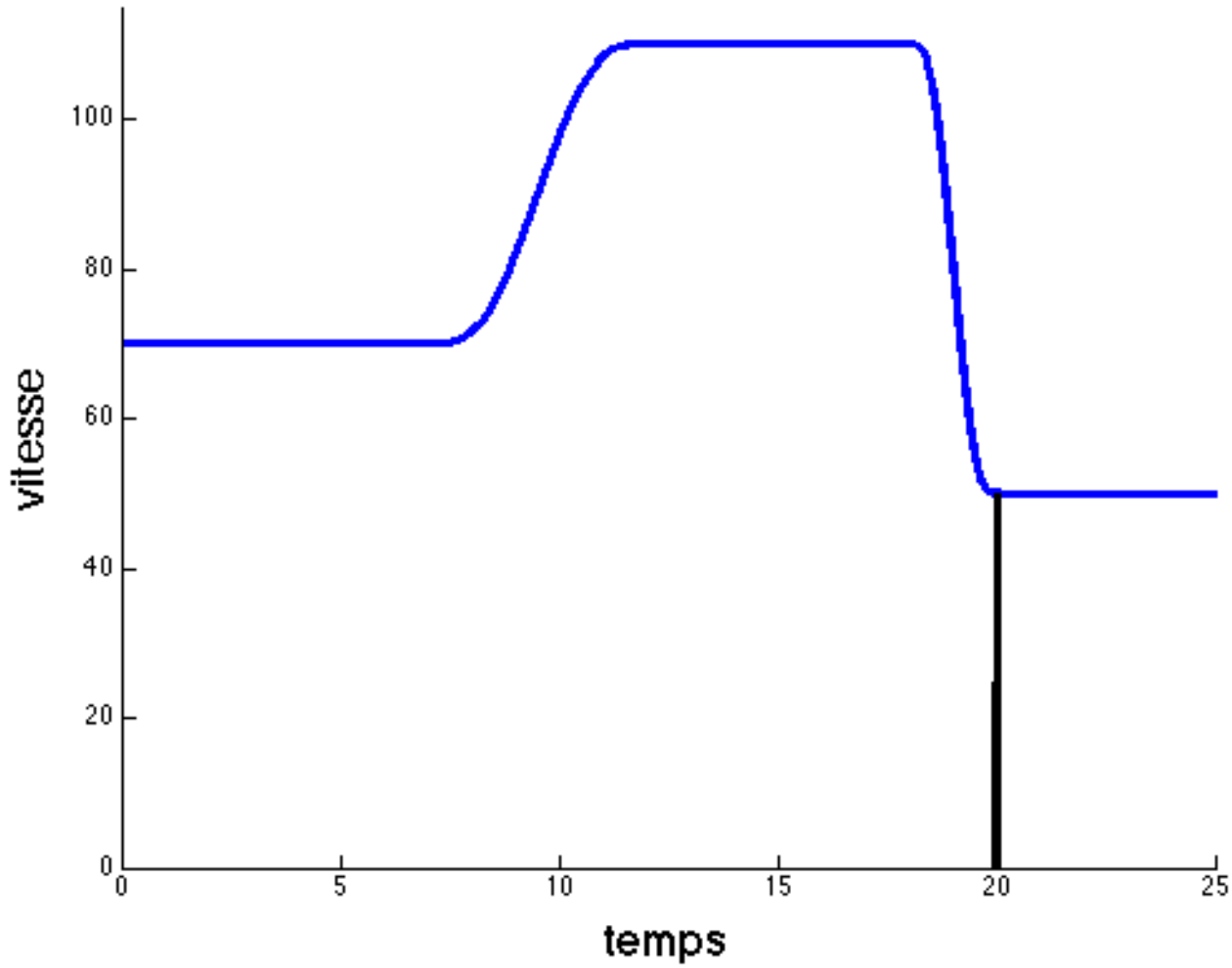
vitesse -> position



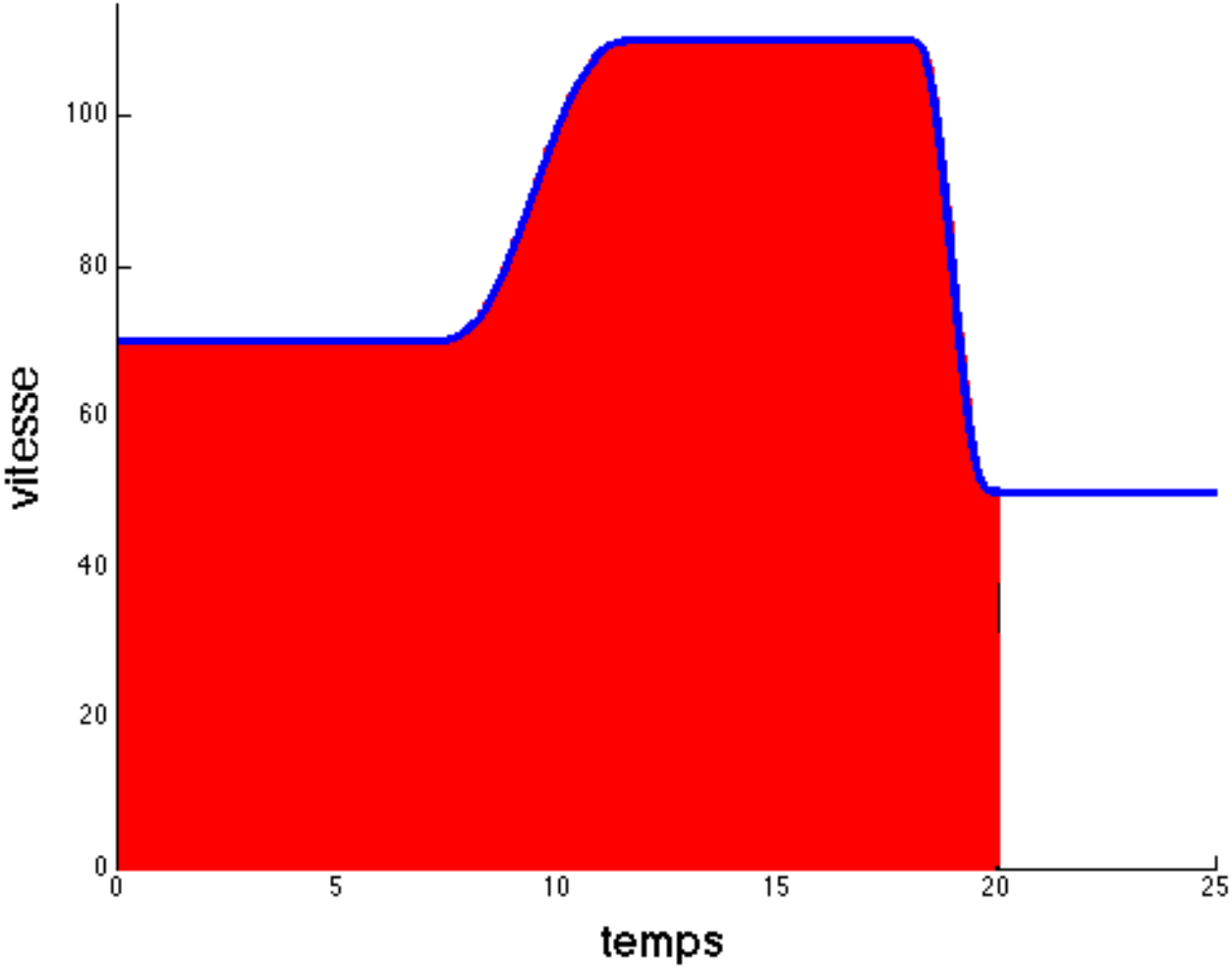
vitesse -> position



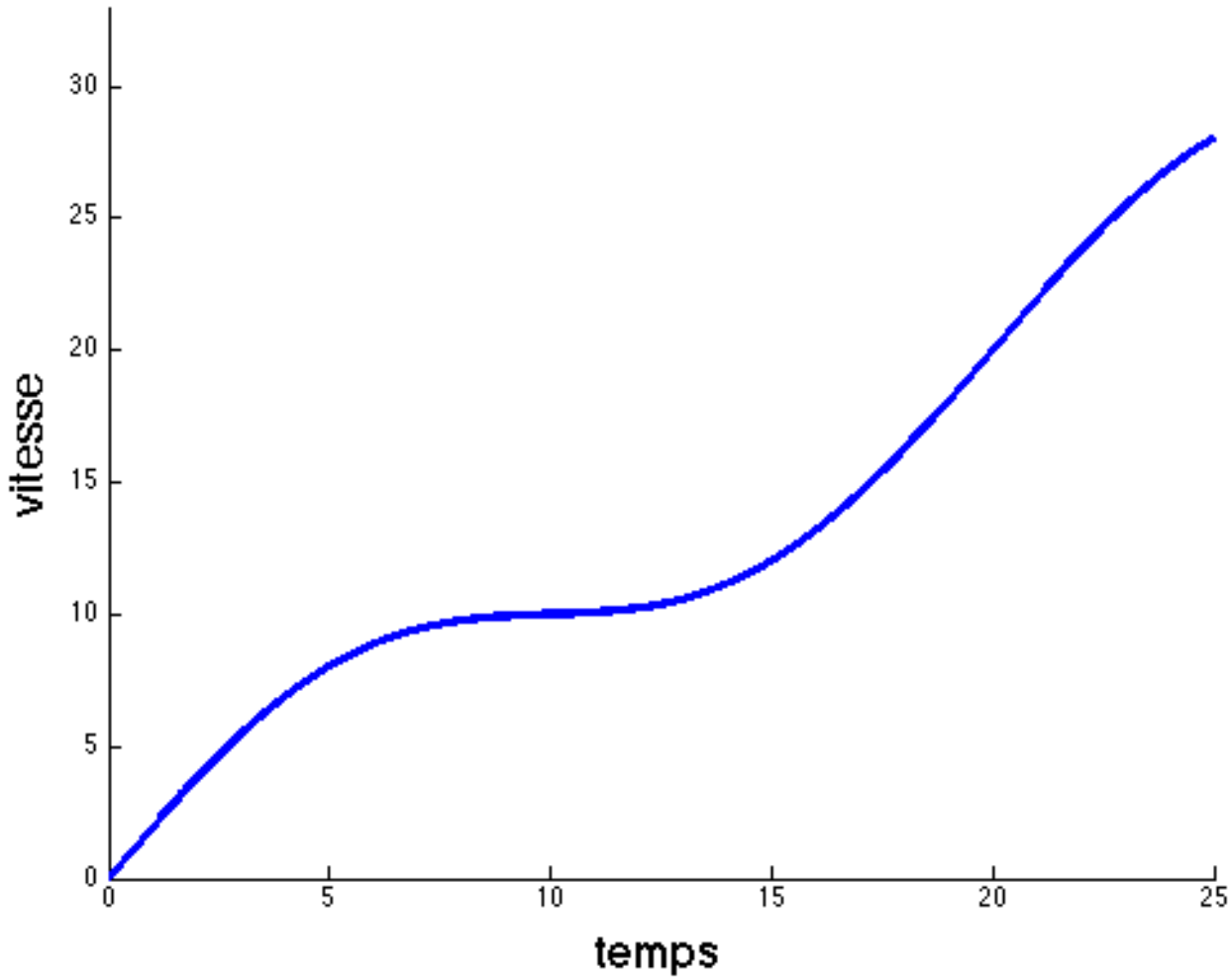
vitesse -> position



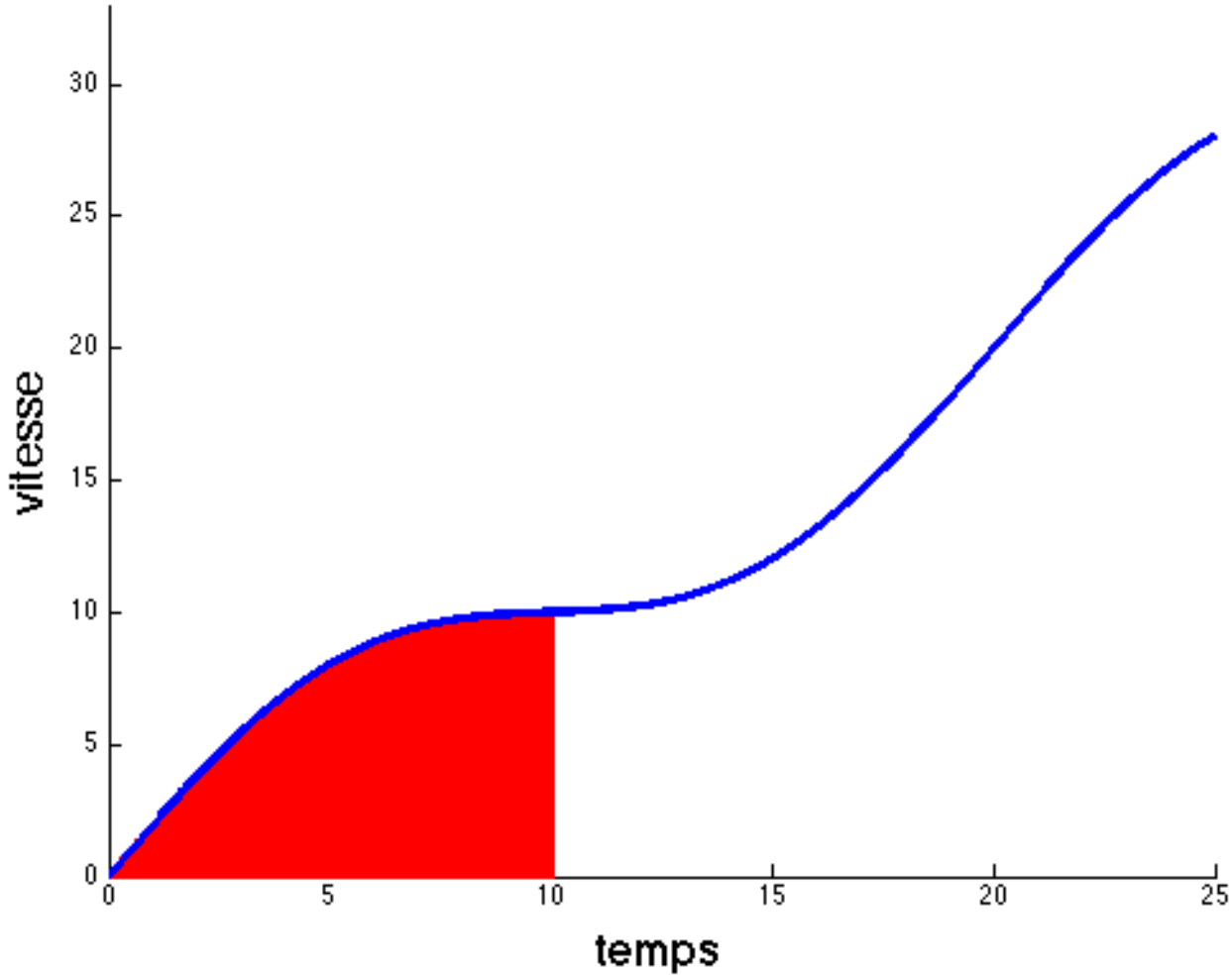
vitesse -> position



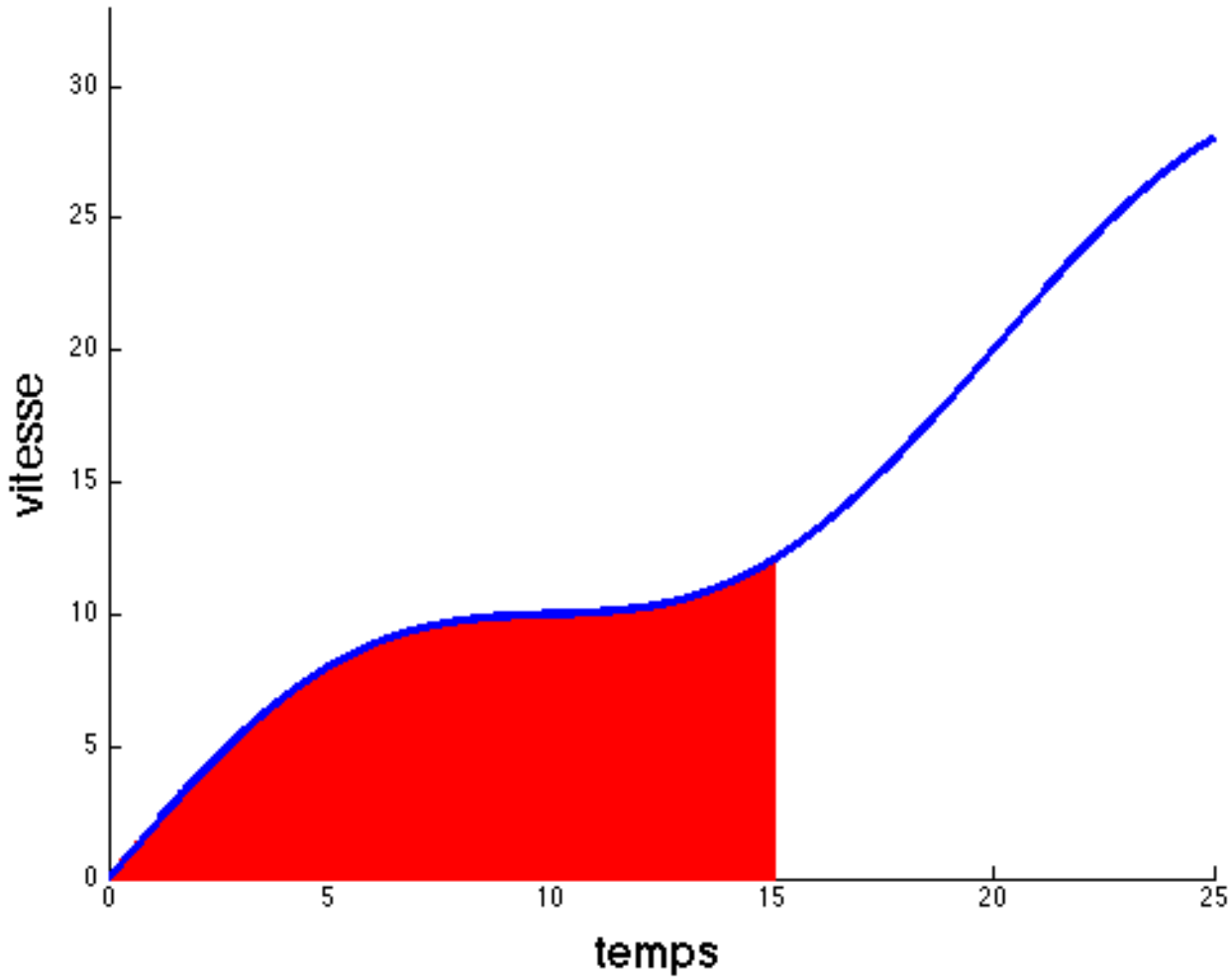
vitesse -> position



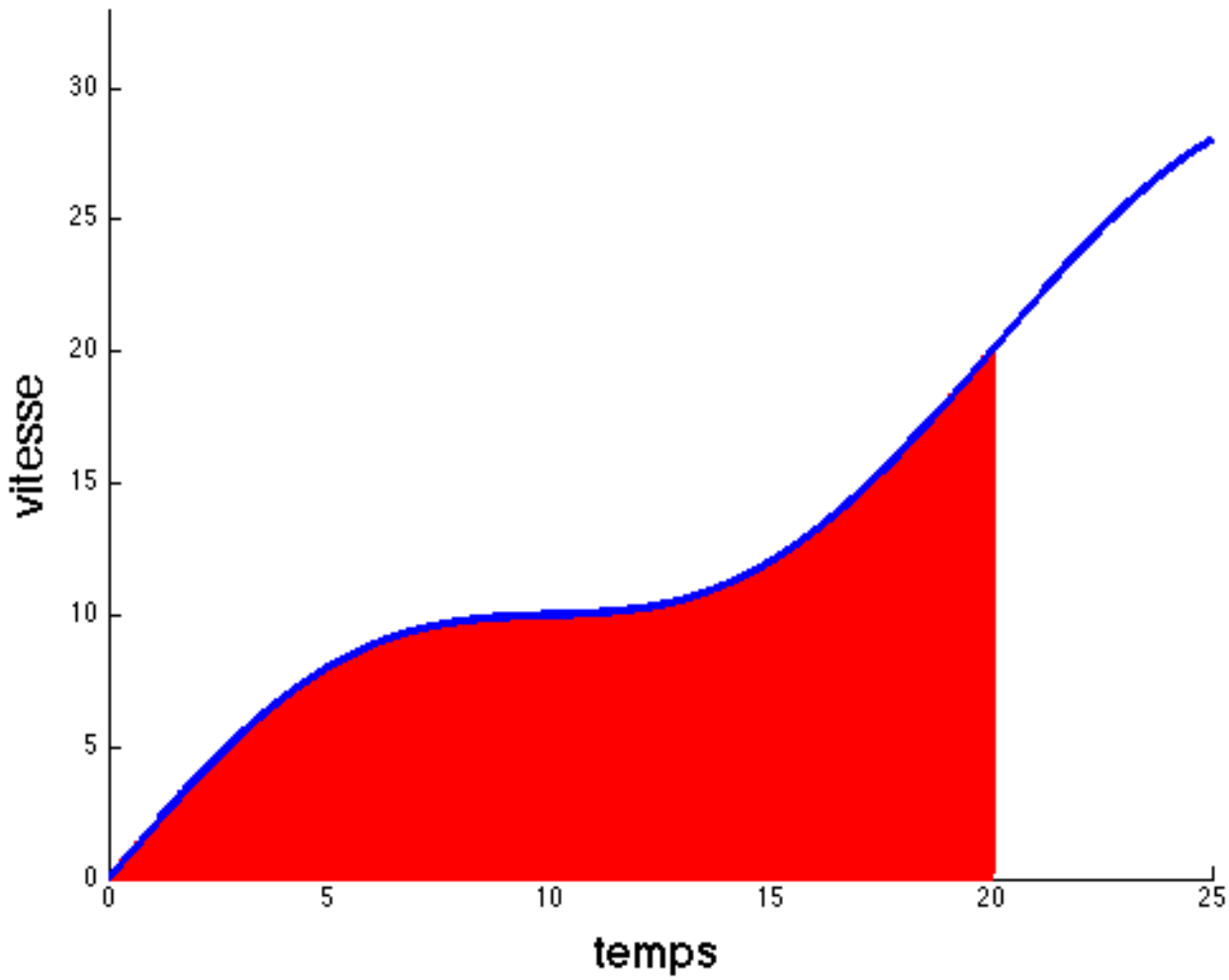
vitesse -> position



vitesse -> position

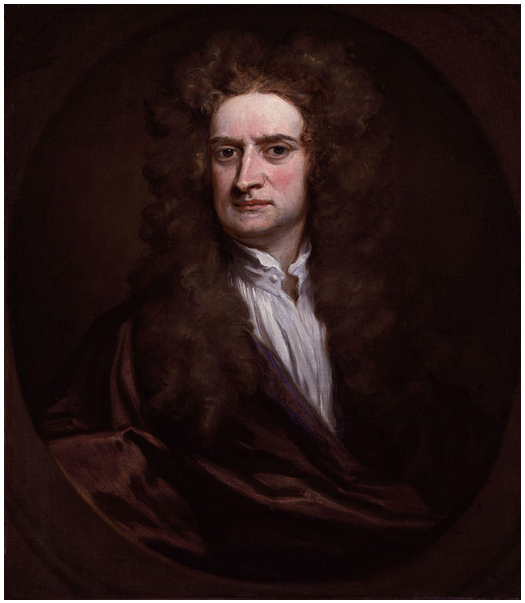


vitesse -> position



Calcul intégral (fin 17e siècle).

Leibniz



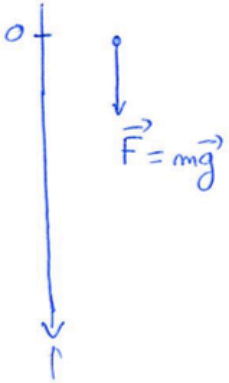
Newton

Règles de calcul.

accélération(t) -> vitesse(t) -> position(t)

Objet en chute libre

Chute d'un solide. $p(0)=0,\ v(0)=0.$



Loi de Newton : $\cancel{m}\, a(t) = \cancel{m}\, g$

Donc $v(t) = v(0) + \int_0^t g(s)\,ds$

$= gt$

Et $p(t) = p(0) + \int_0^t v(s)\,ds$

$= 0 + \int_0^t gs\,ds$

$$\boxed{p(t) = \frac{gt^2}{2}}$$

Objet en chute libre

Chute d'un solide. $p(0)=0,\ v(0)=0.$

Loi de Newton : $\cancel{m}\, a(t) = \cancel{m}\, g$

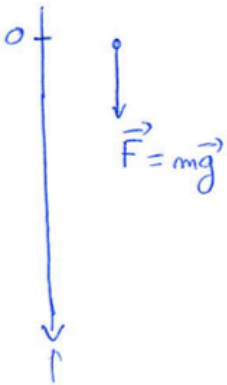
Donc $v(t) = v(0) + \int_0^t g(s)\,ds$

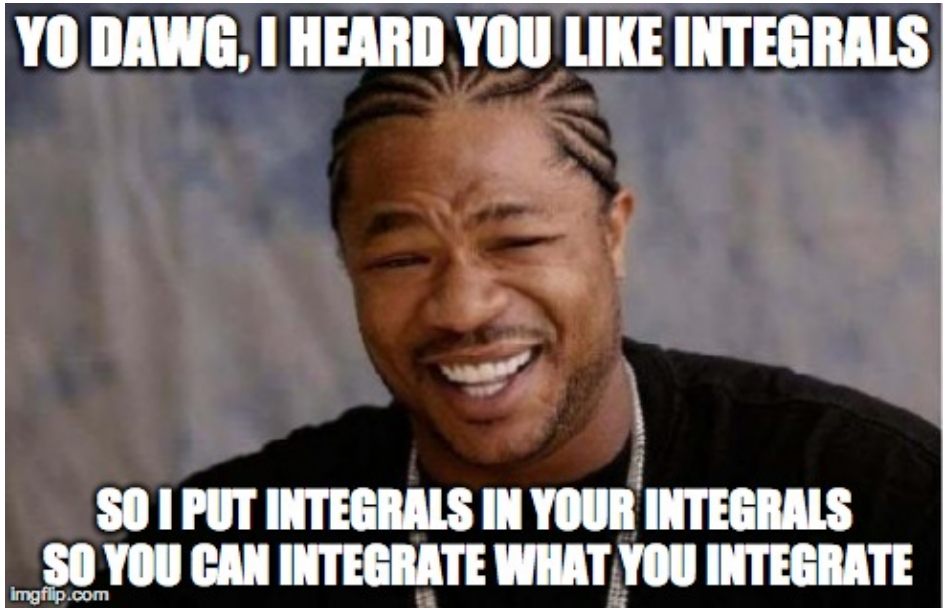
$= gt$

Et $p(t) = p(0) + \int_0^t v(s)\,ds$

$= 0 + \int_0^t gs\,ds$

$$\boxed{p(t) = \frac{gt^2}{2}}$$

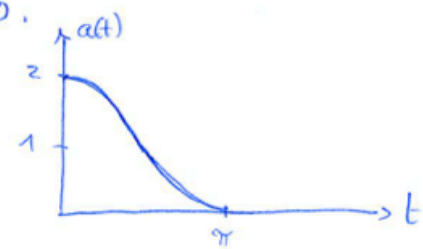




Accélération localisée, dans le vide

Coup d'accélération. $p(0)=0,\ v(0)=0.$

- $$a(t) = \begin{cases} \cos t + 1 & \text{pour } t \leq \pi \\ 0 & \text{pour } t \geq \pi \end{cases}$$

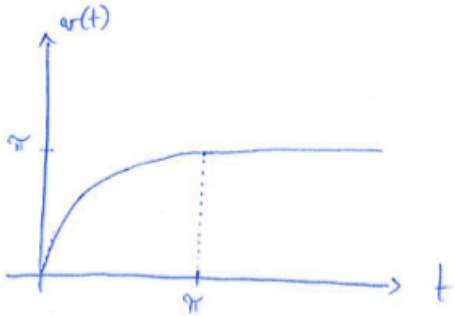


- Pour $t \leq \pi$,

$$v(t) = 0 + \int_0^t \cos s + 1 \, ds$$

$$= \sin t + t.$$

Pour $t \geq \pi$, $v(t) = v(\pi) = \pi$.



- Pour $t \leq \pi$,

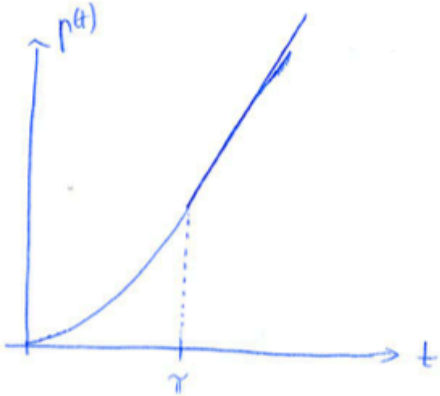
$$p(t) = 0 + \int_0^t \sin s + s \, ds$$

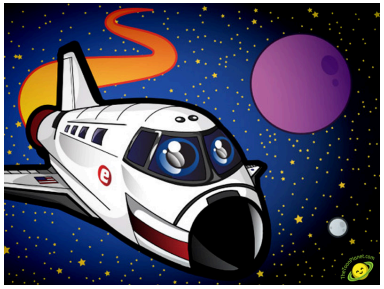
$$= 1 - \cos t + \frac{t^2}{2}$$

Pour $t \geq \pi$,

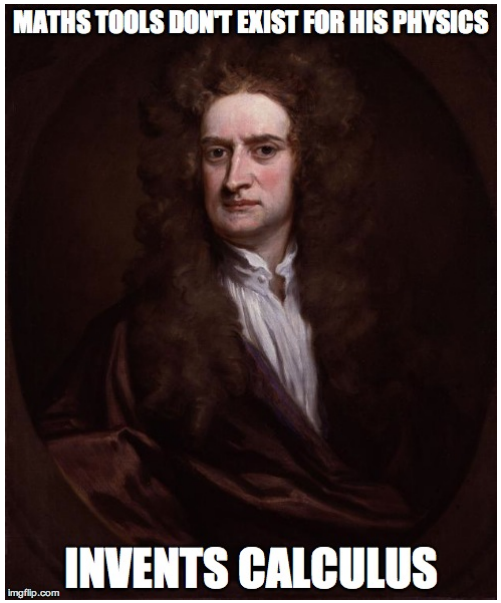
$$p(t) = p(\pi) + v(\pi)t$$

$$= \left(2 + \frac{\pi^2}{2}\right) + \pi t.$$

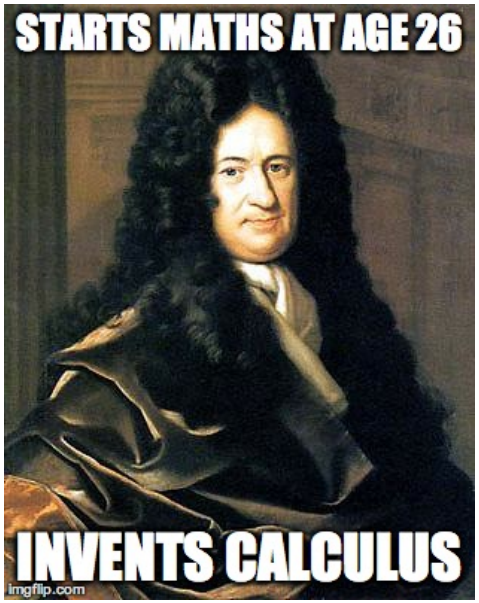




Calcul différentiel et intégral



Newton

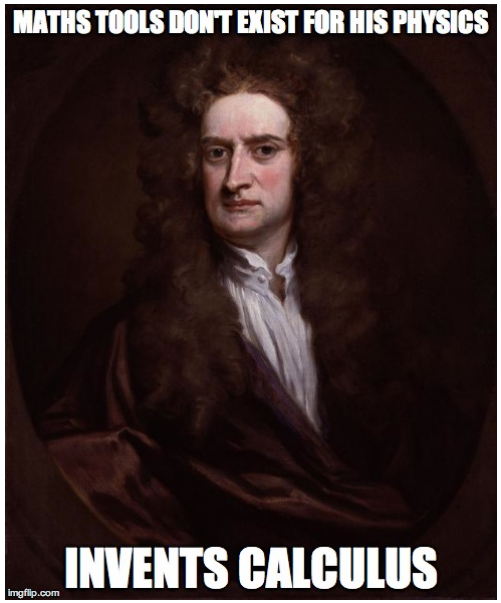


Leibniz

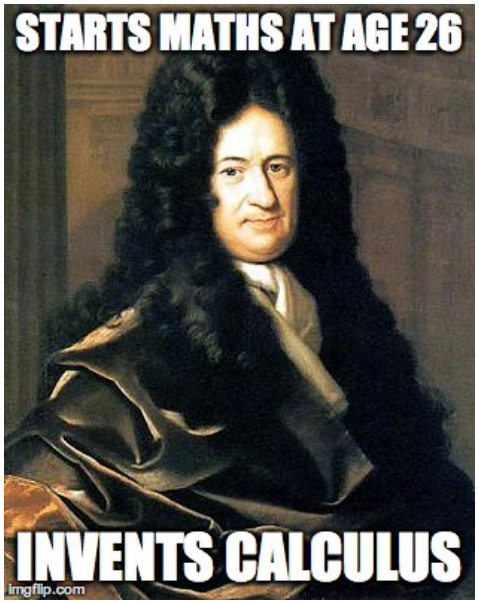
Différentier : p(t) -> v(t) -> a(t)

Intégrer : a(t) -> v(t) -> p(t)

Calcul différentiel et intégral



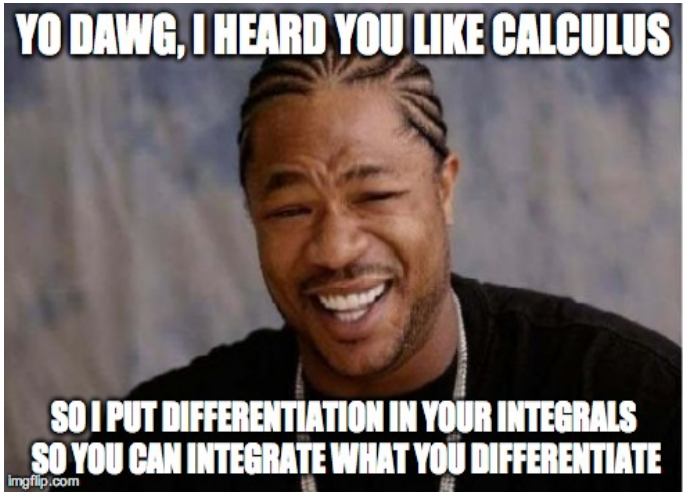
Newton



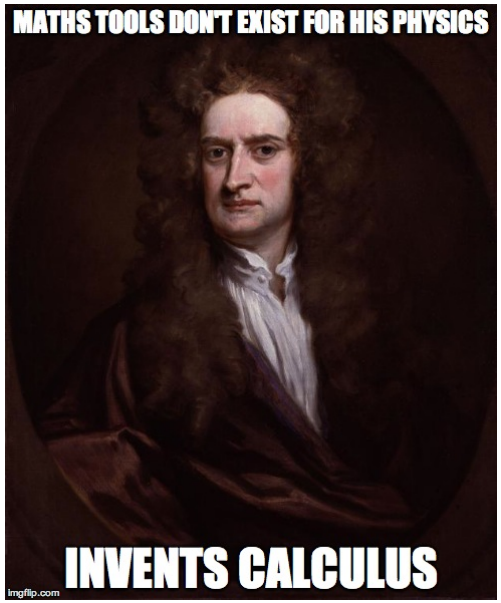
Leibniz

Différentier : p(t) -> v(t) -> a(t)

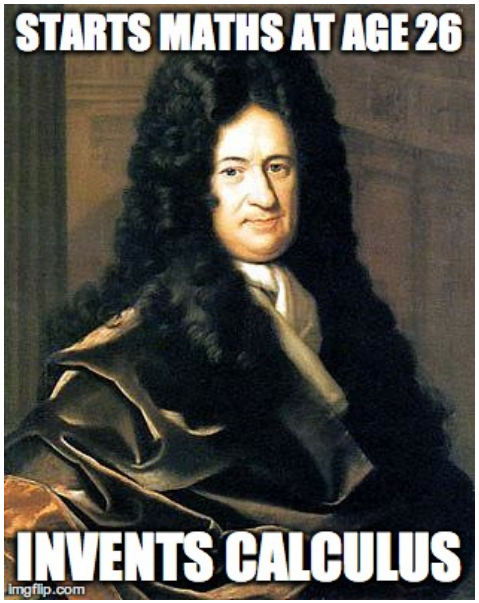
Intégrer : a(t) -> v(t) -> p(t)



Calcul différentiel et intégral



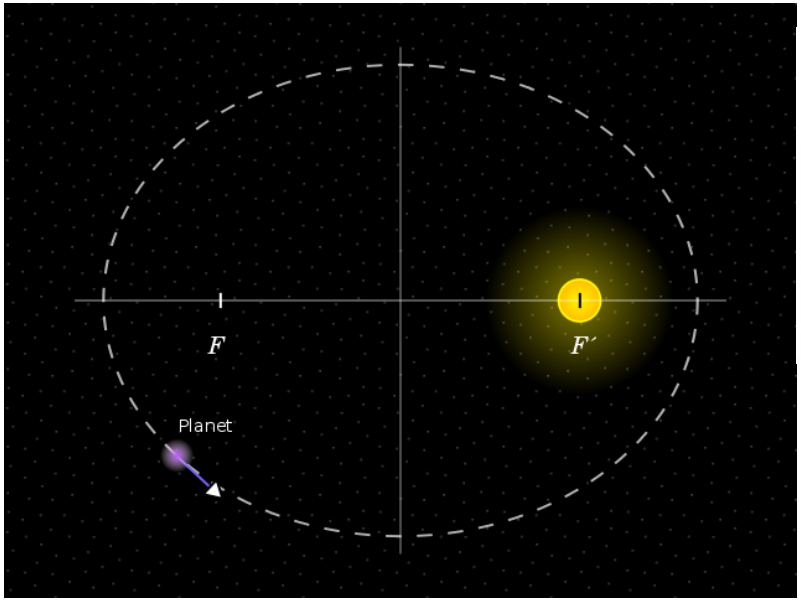
Newton

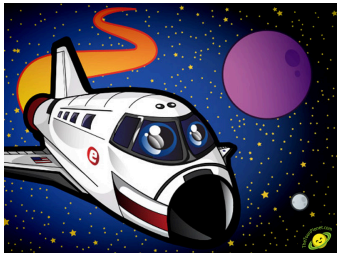


Leibniz

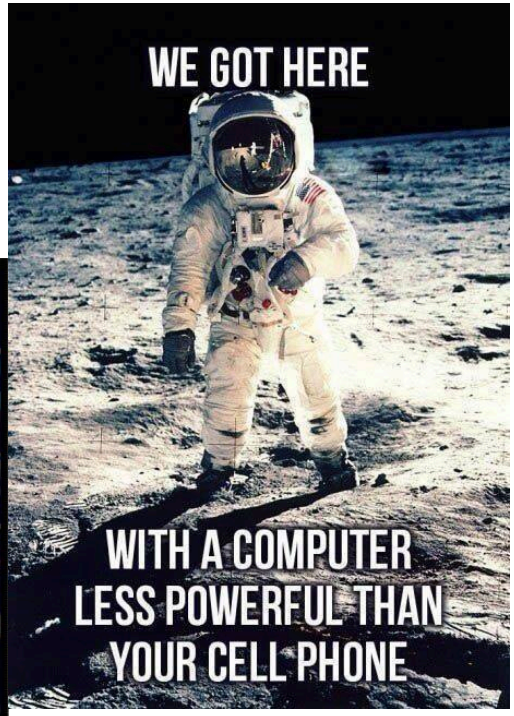
Différentier : p(t) -> v(t) -> a(t)

Intégrer : a(t) -> v(t) -> p(t)


F
F′
Planet



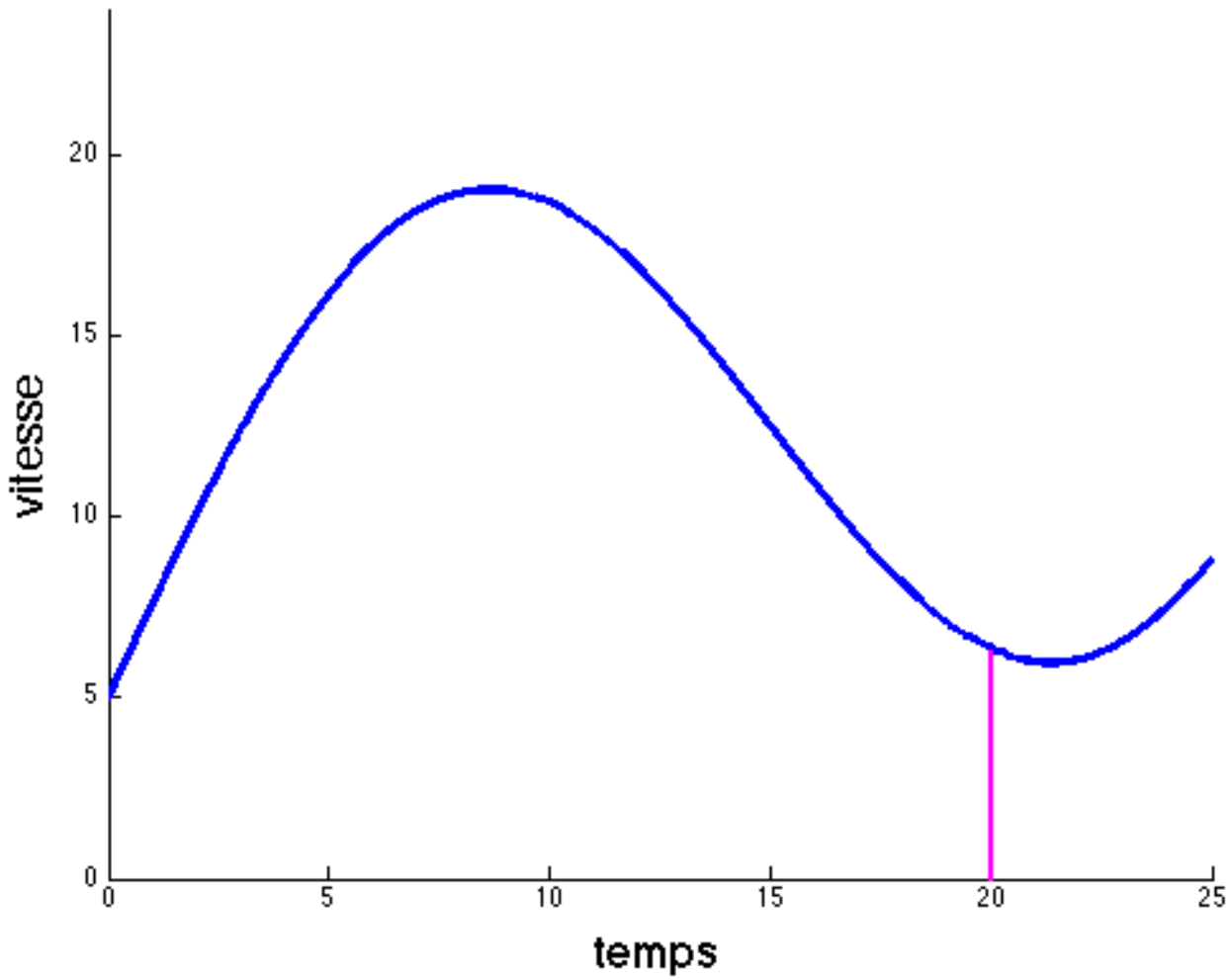
Science Rocks


WE GOT HERE
WITH A COMPUTER
LESS POWERFUL THAN
YOUR CELL PHONE

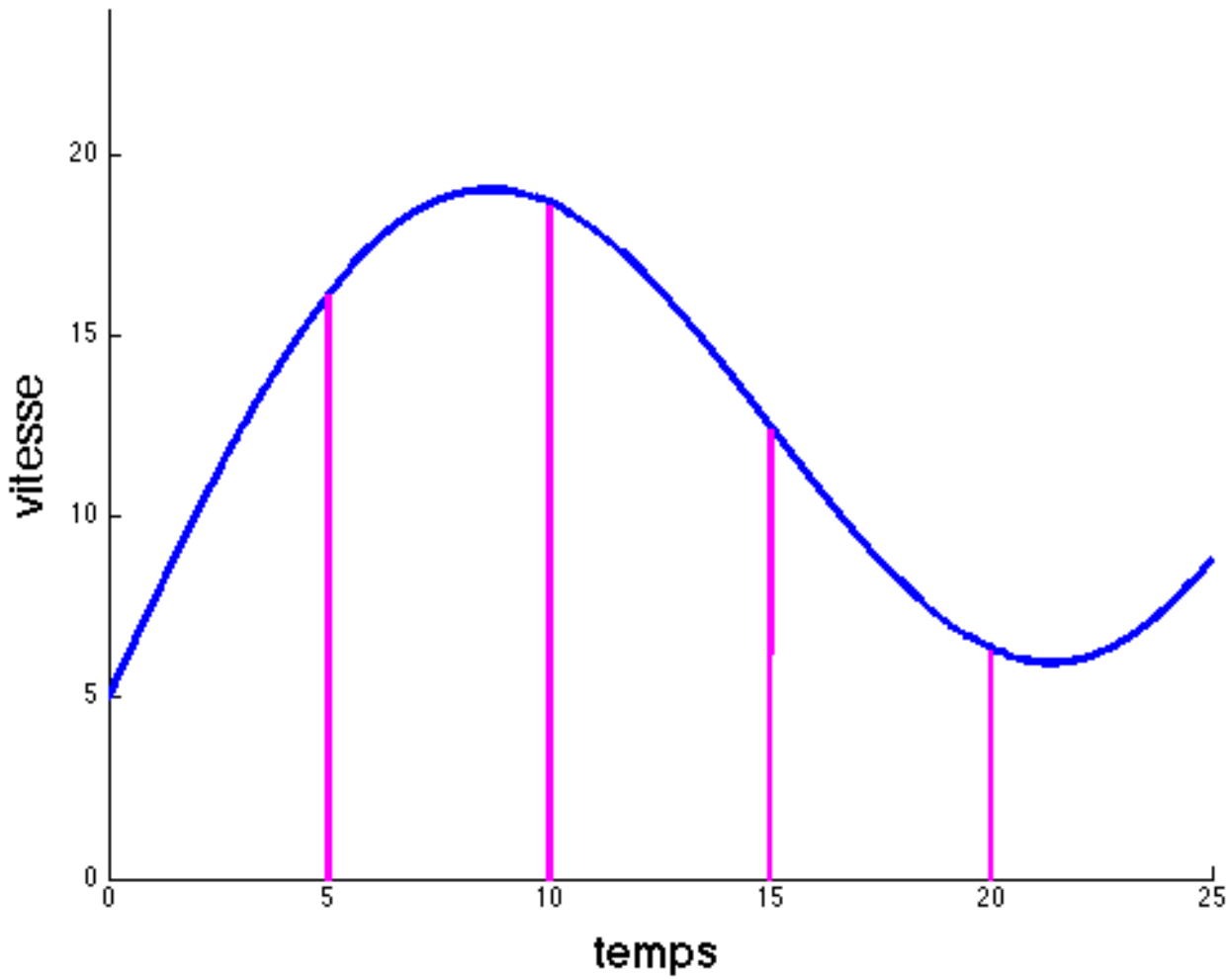
Cas sans formule

$$I(t) = \int_0^t e^{-\frac{u^2}{2}} \, du = ?$$

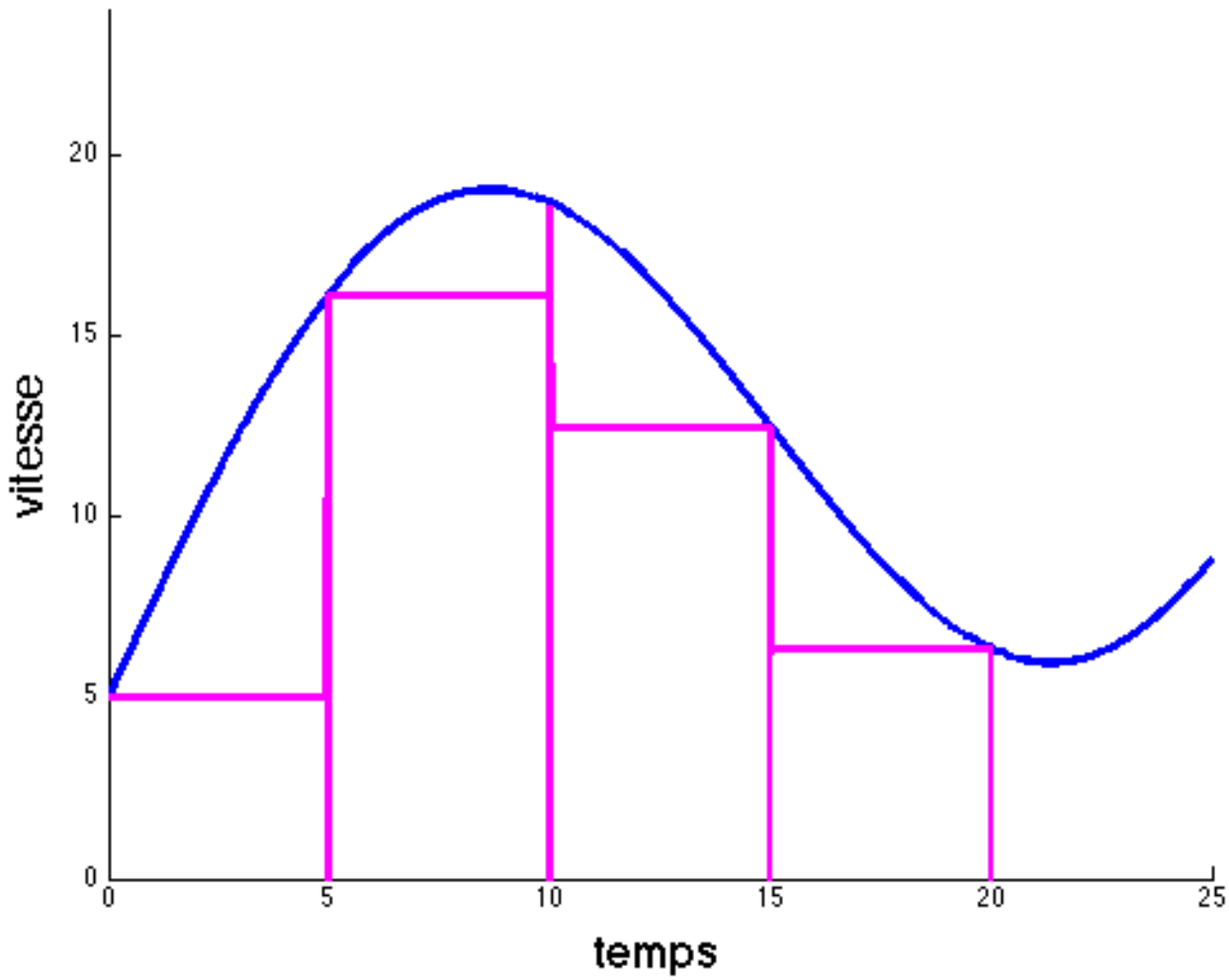
La méthode des rectangles



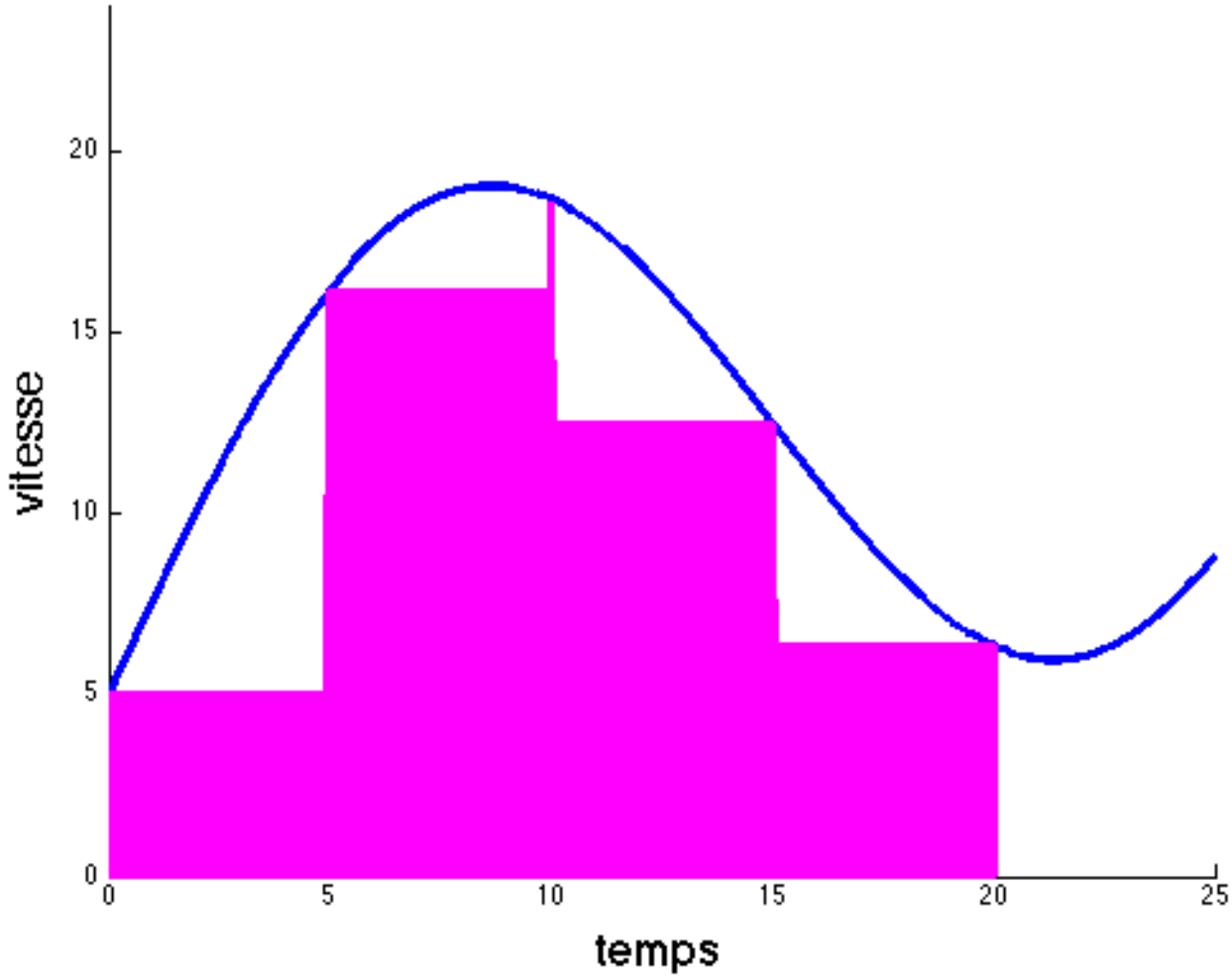
La méthode des rectangles



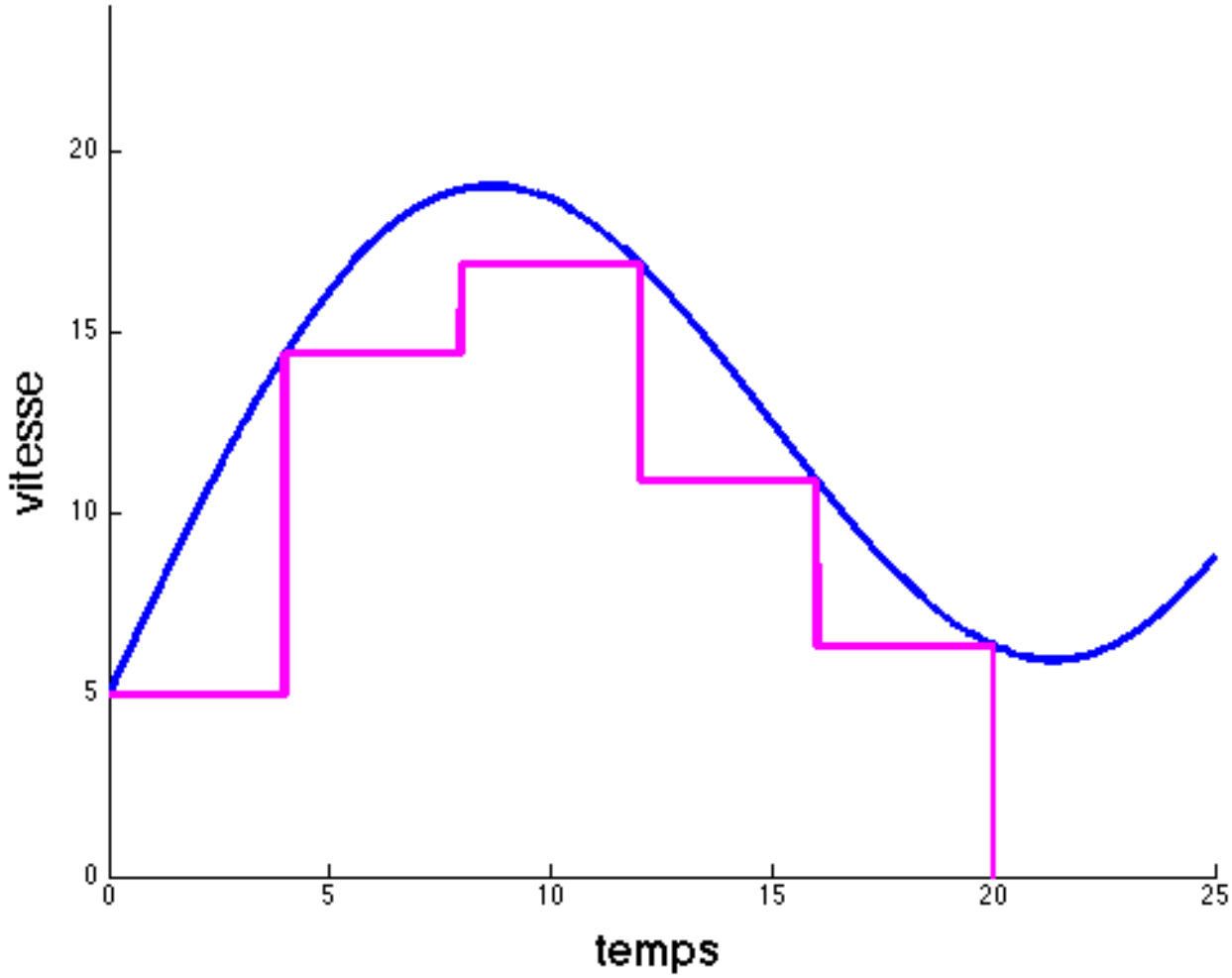
La méthode des rectangles



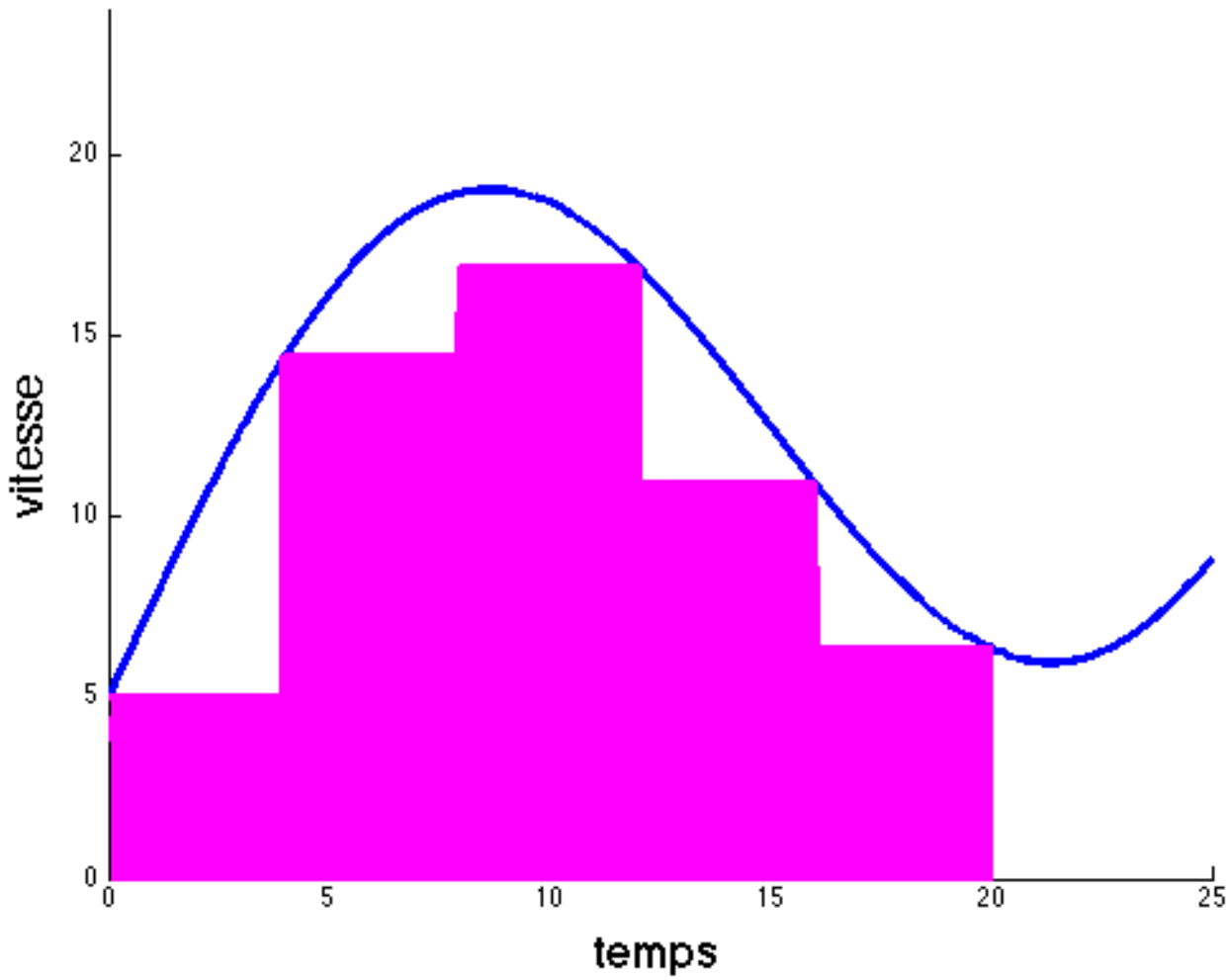
La méthode des rectangles



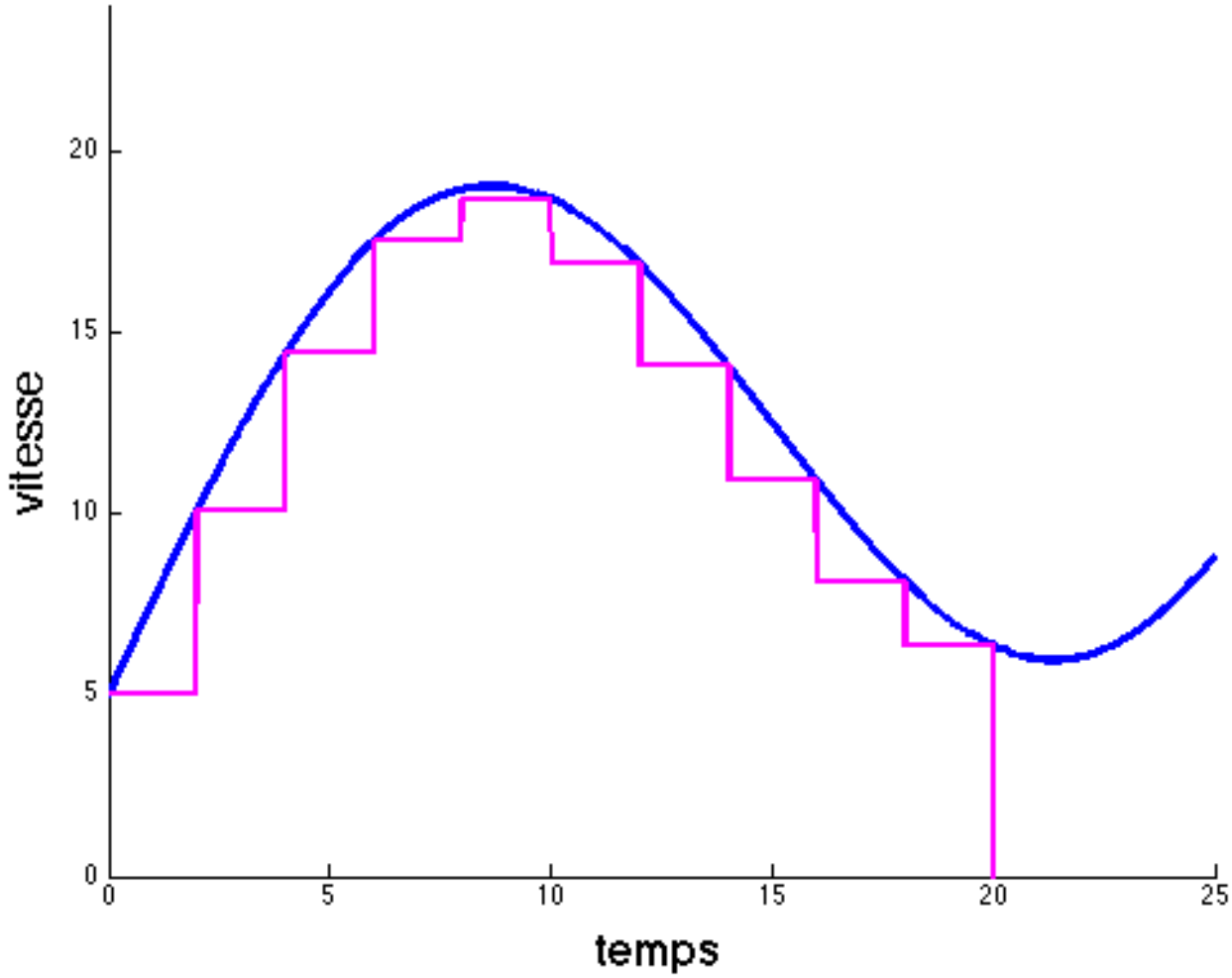
La méthode des rectangles



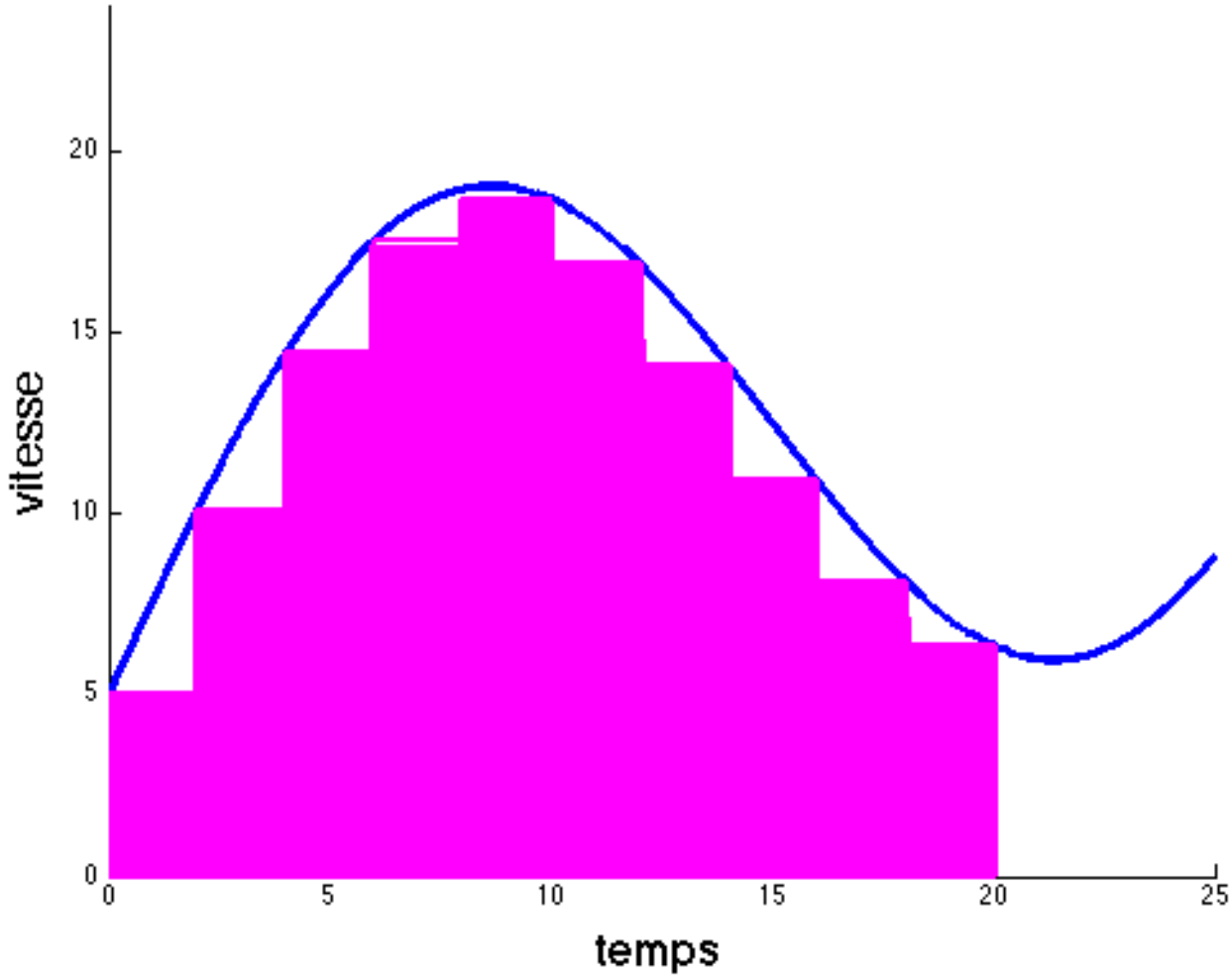
La méthode des rectangles



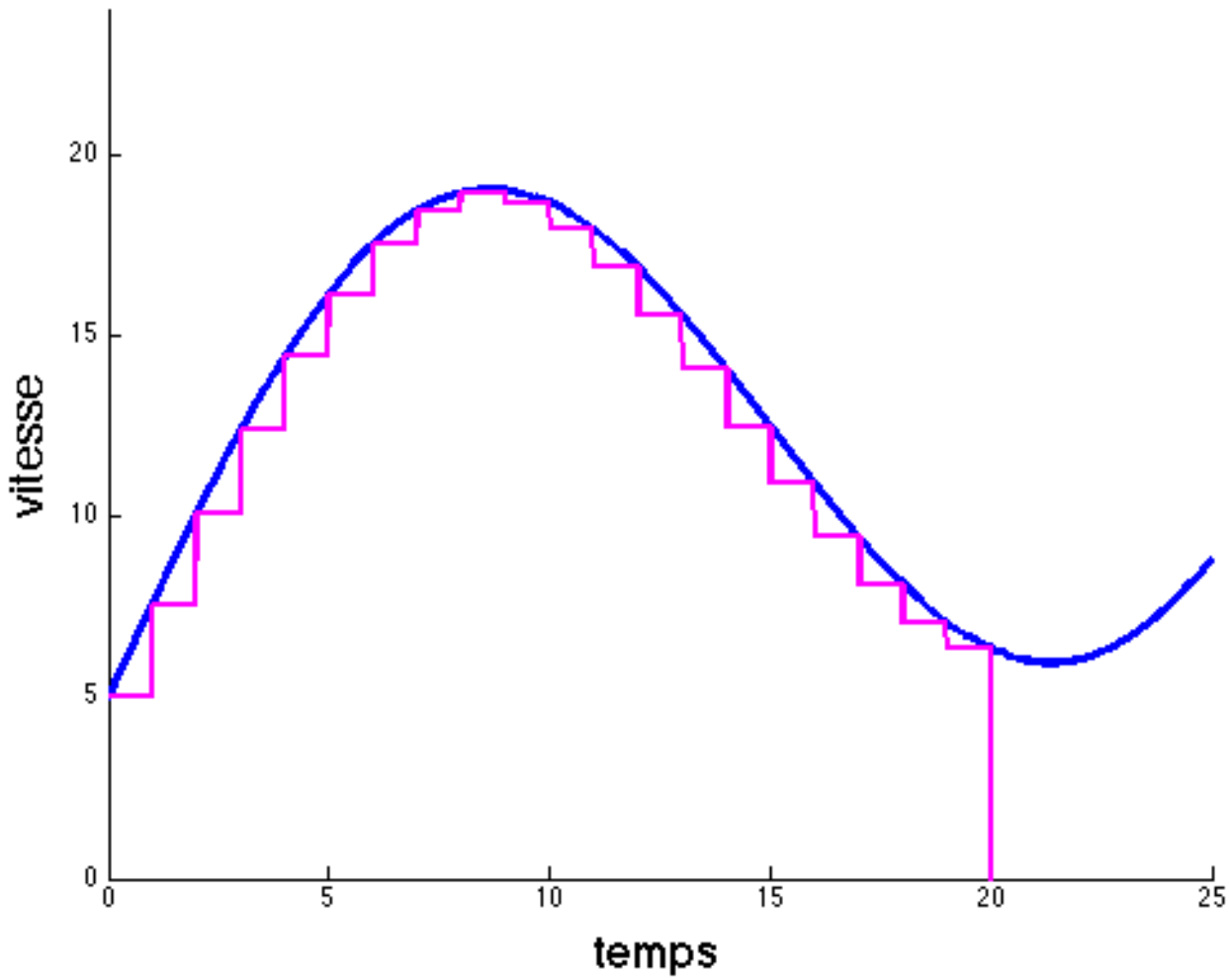
La méthode des rectangles



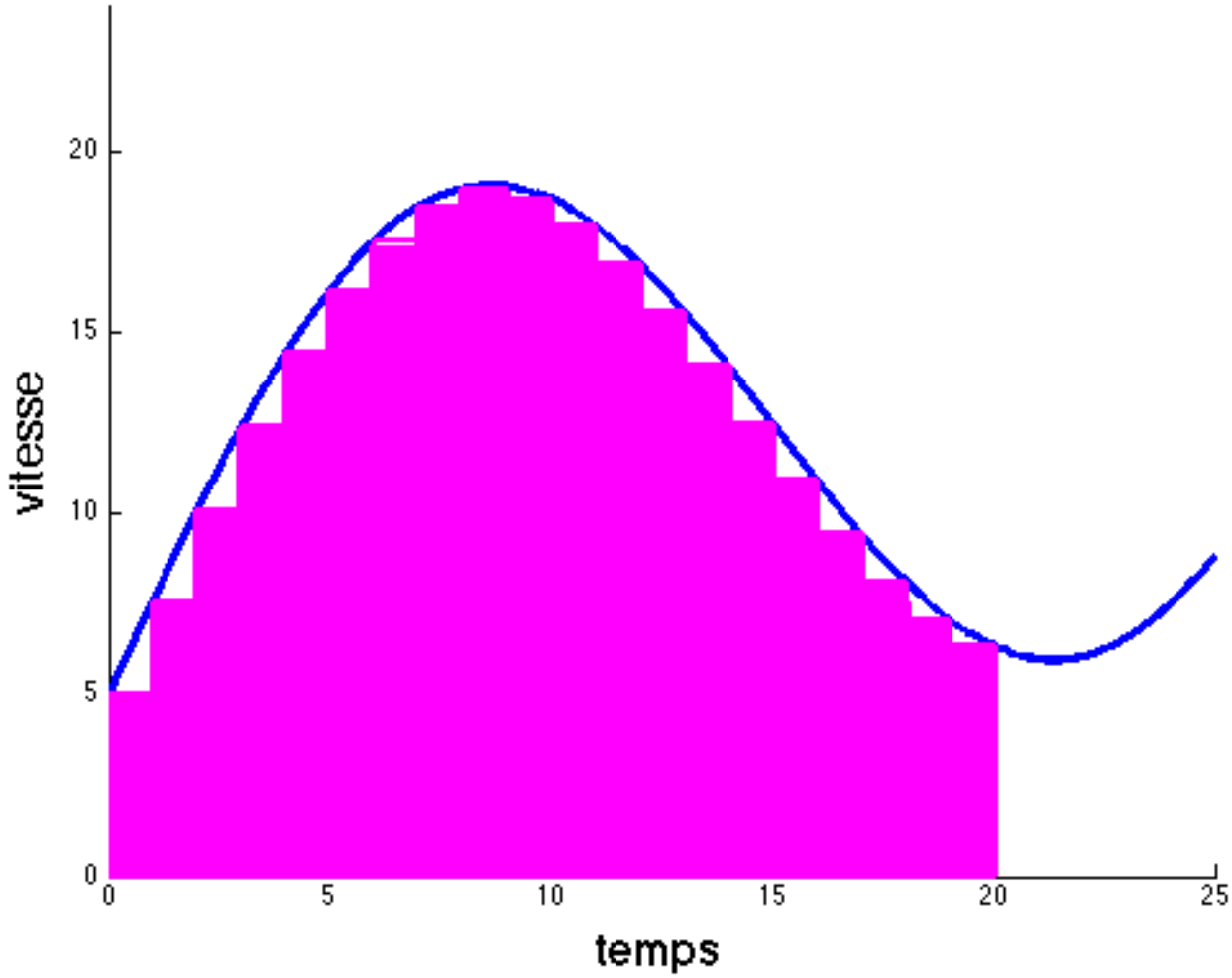
La méthode des rectangles



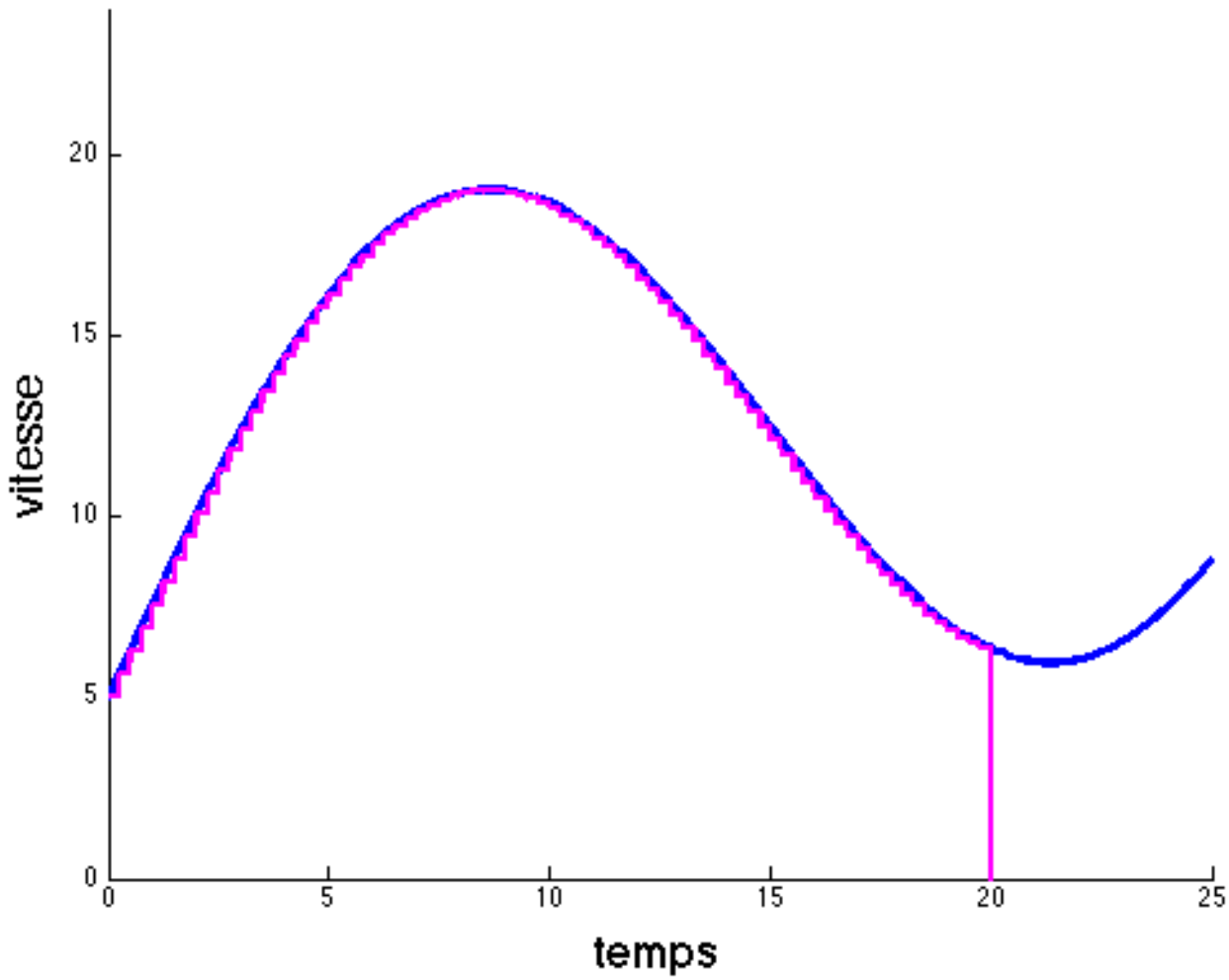
La méthode des rectangles



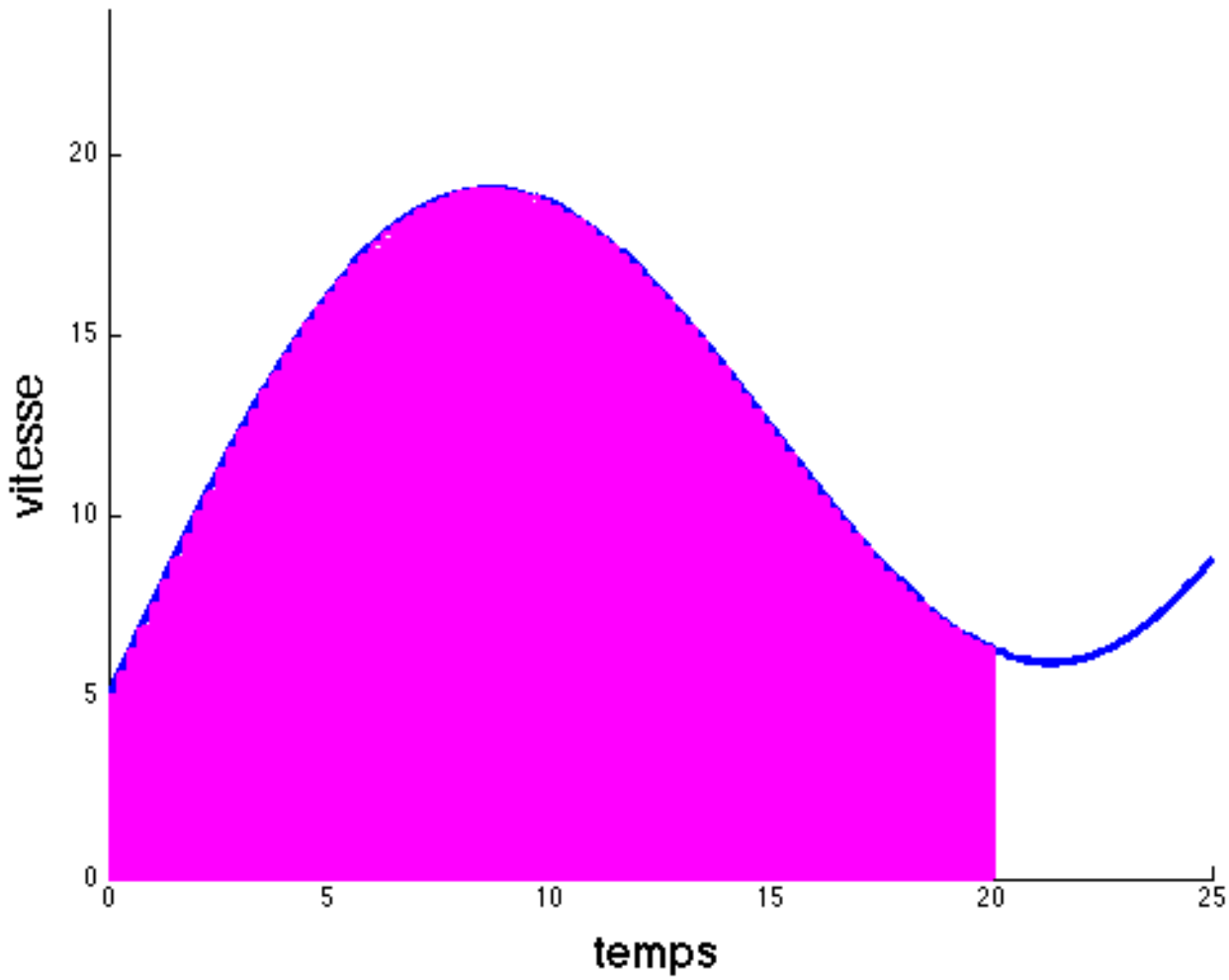
La méthode des rectangles



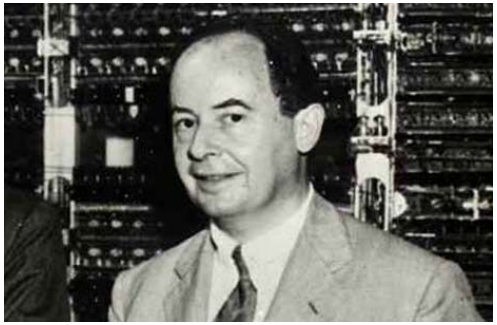
La méthode des rectangles

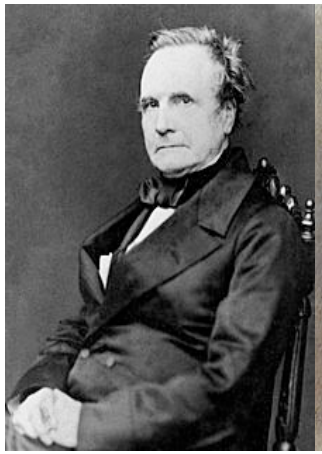


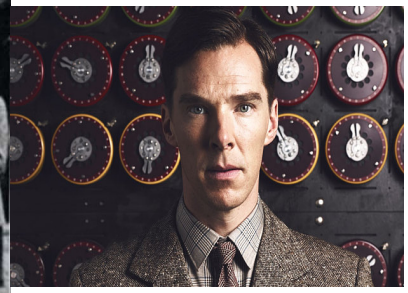
La méthode des rectangles



Computer = calculateur

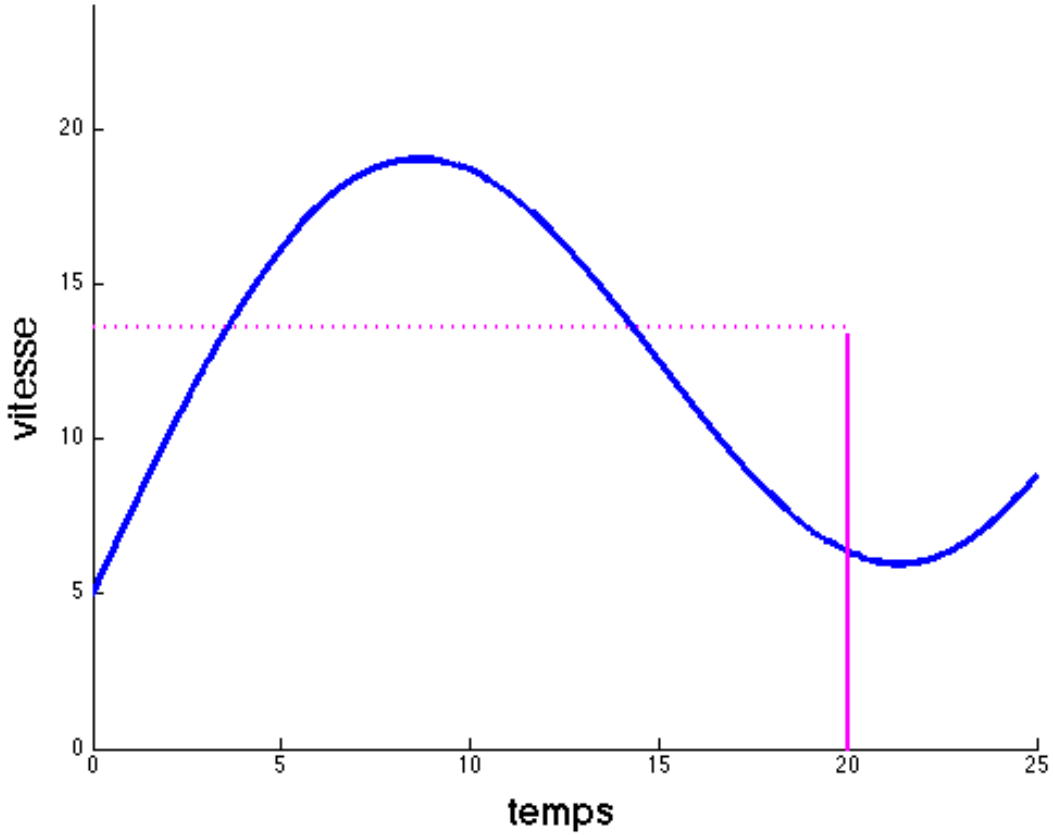




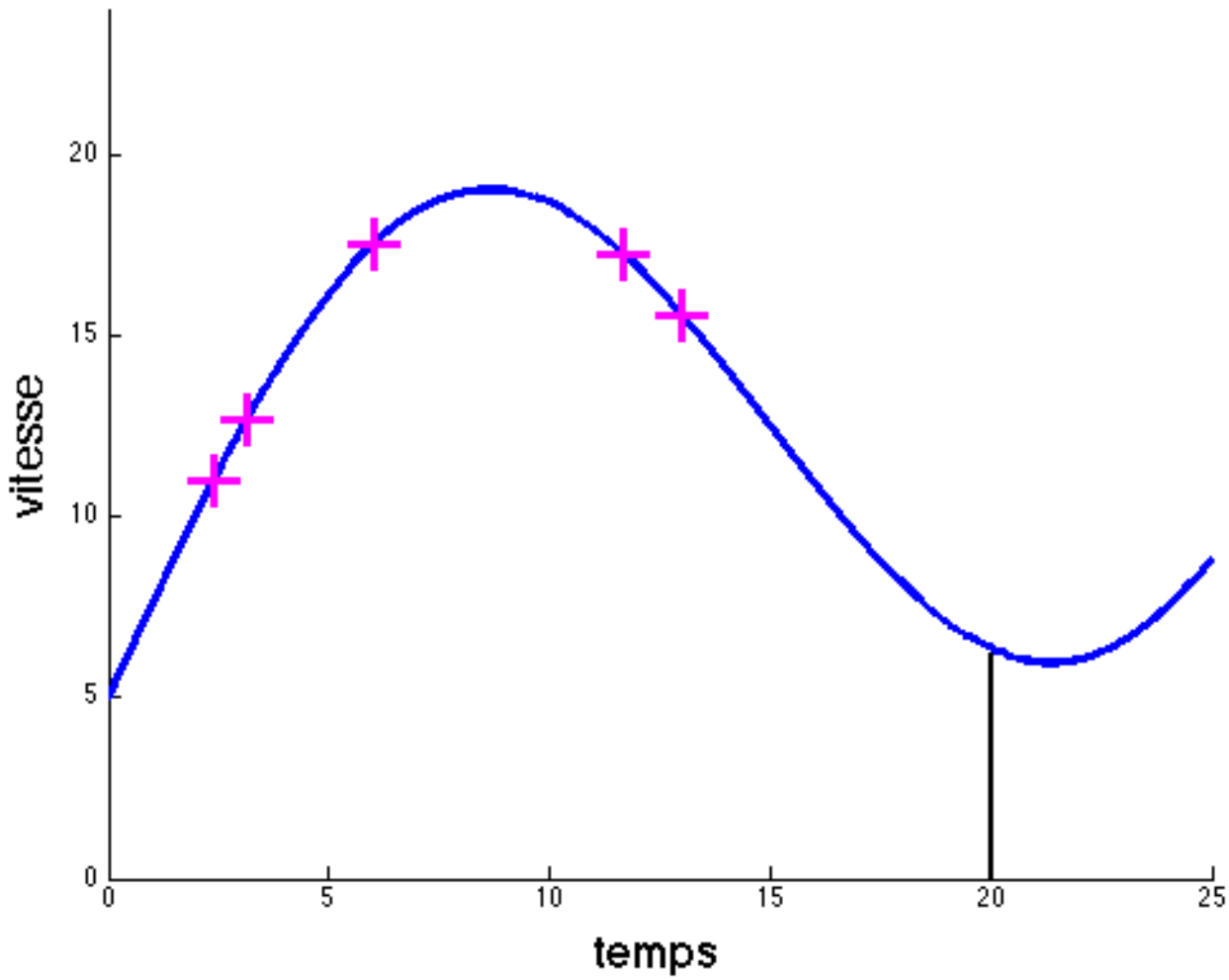


Une méthode numérique … probabiliste

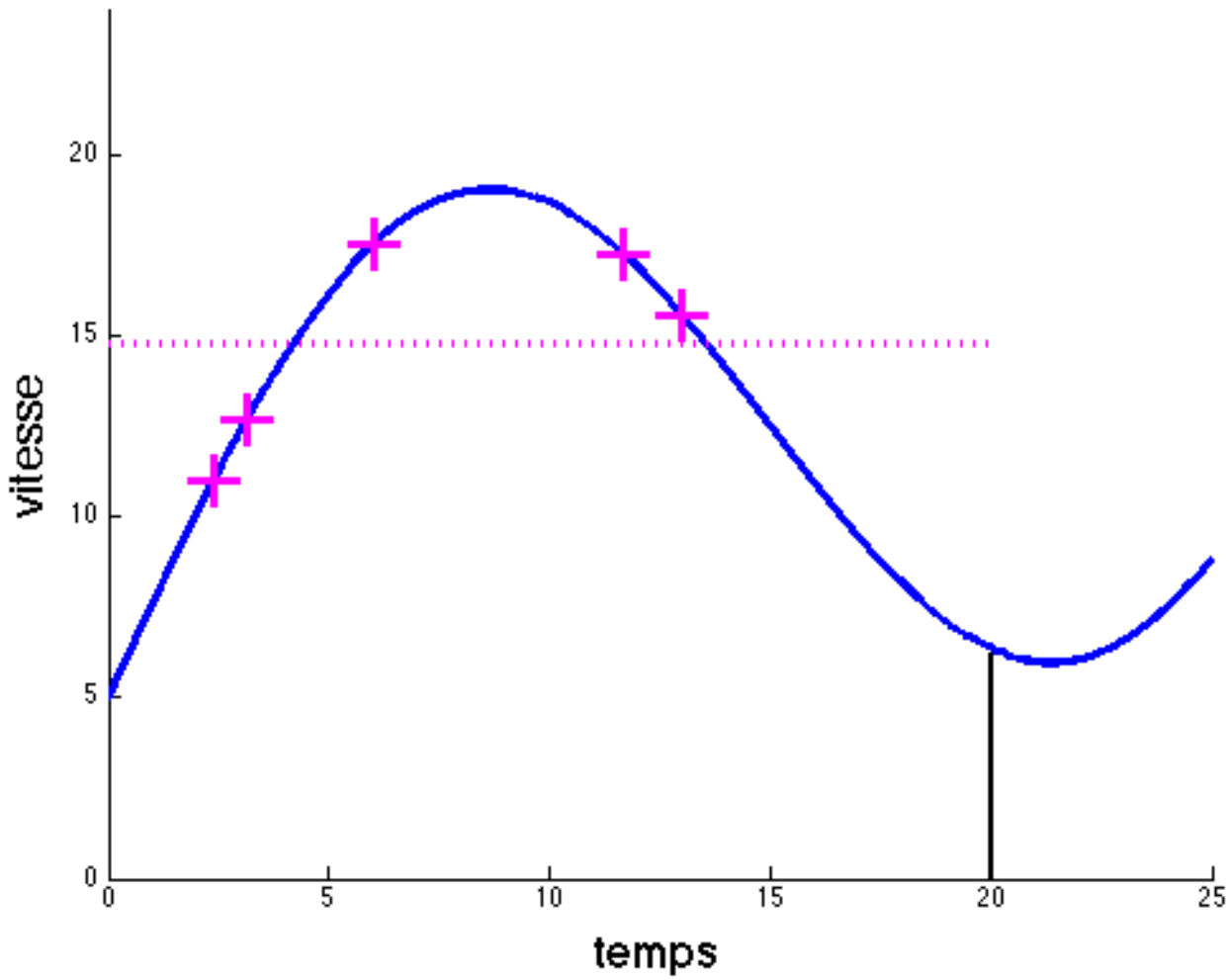
Distance totale = vitesse moyenne×temps total



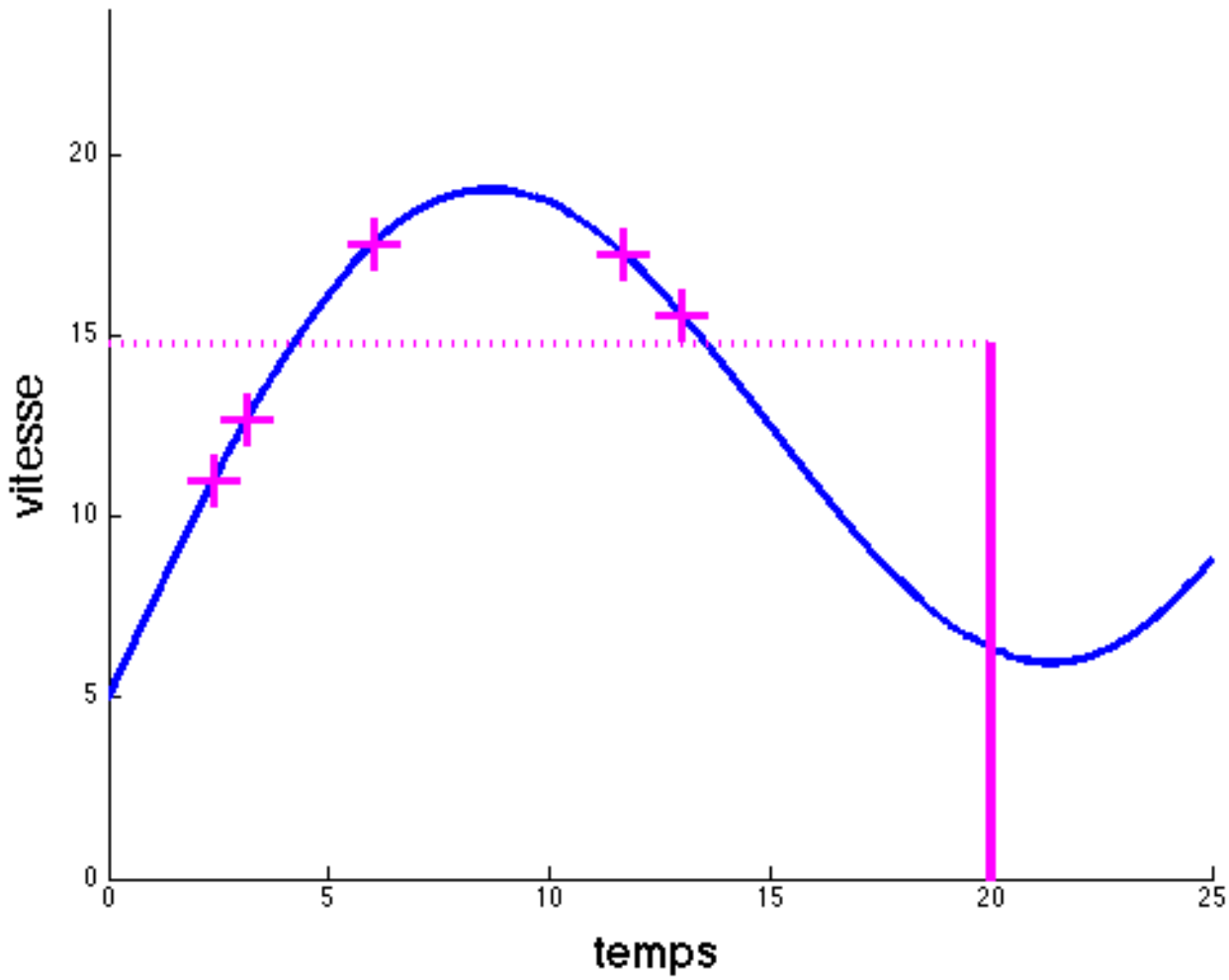
La méthode Monte Carlo



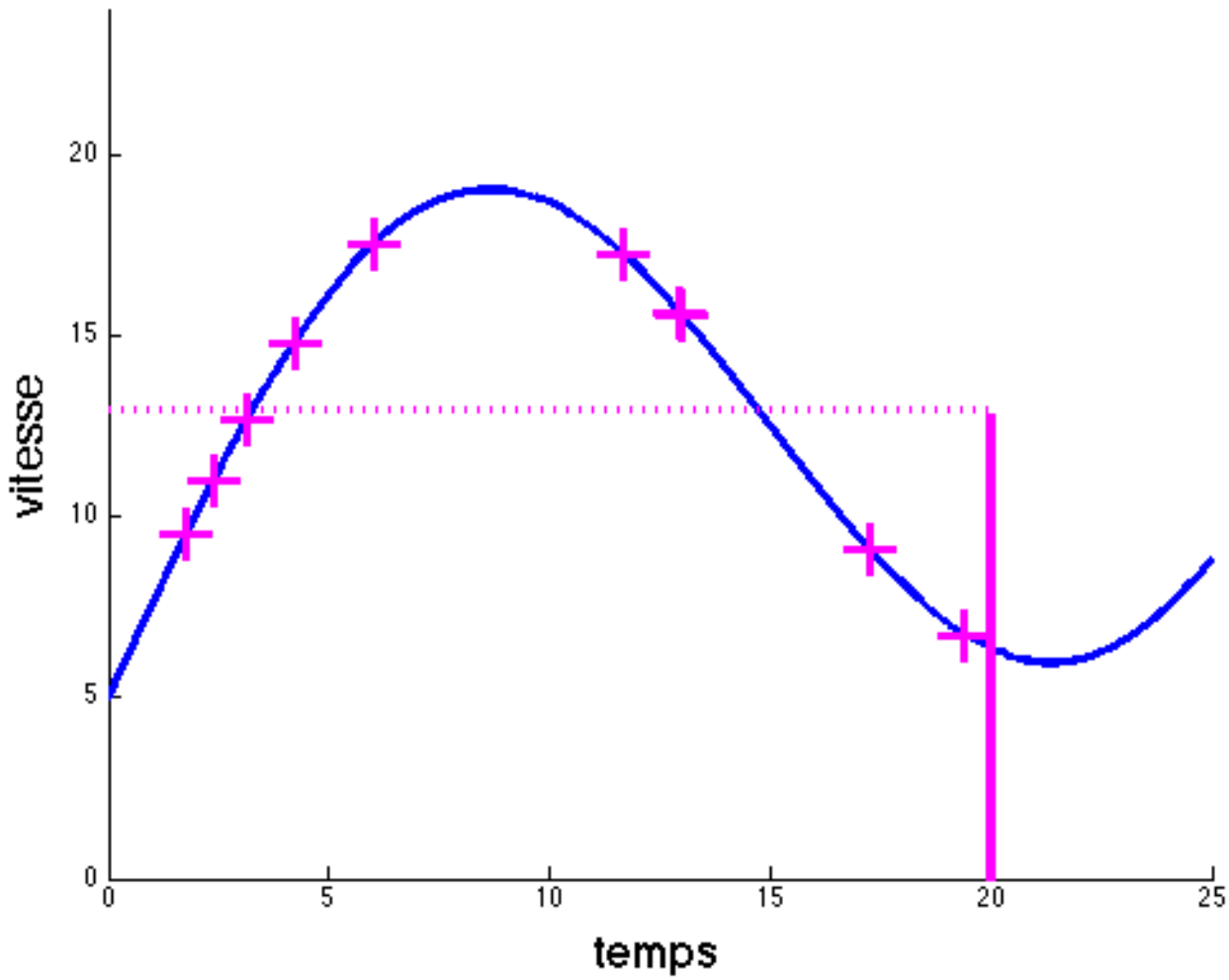
La méthode Monte Carlo



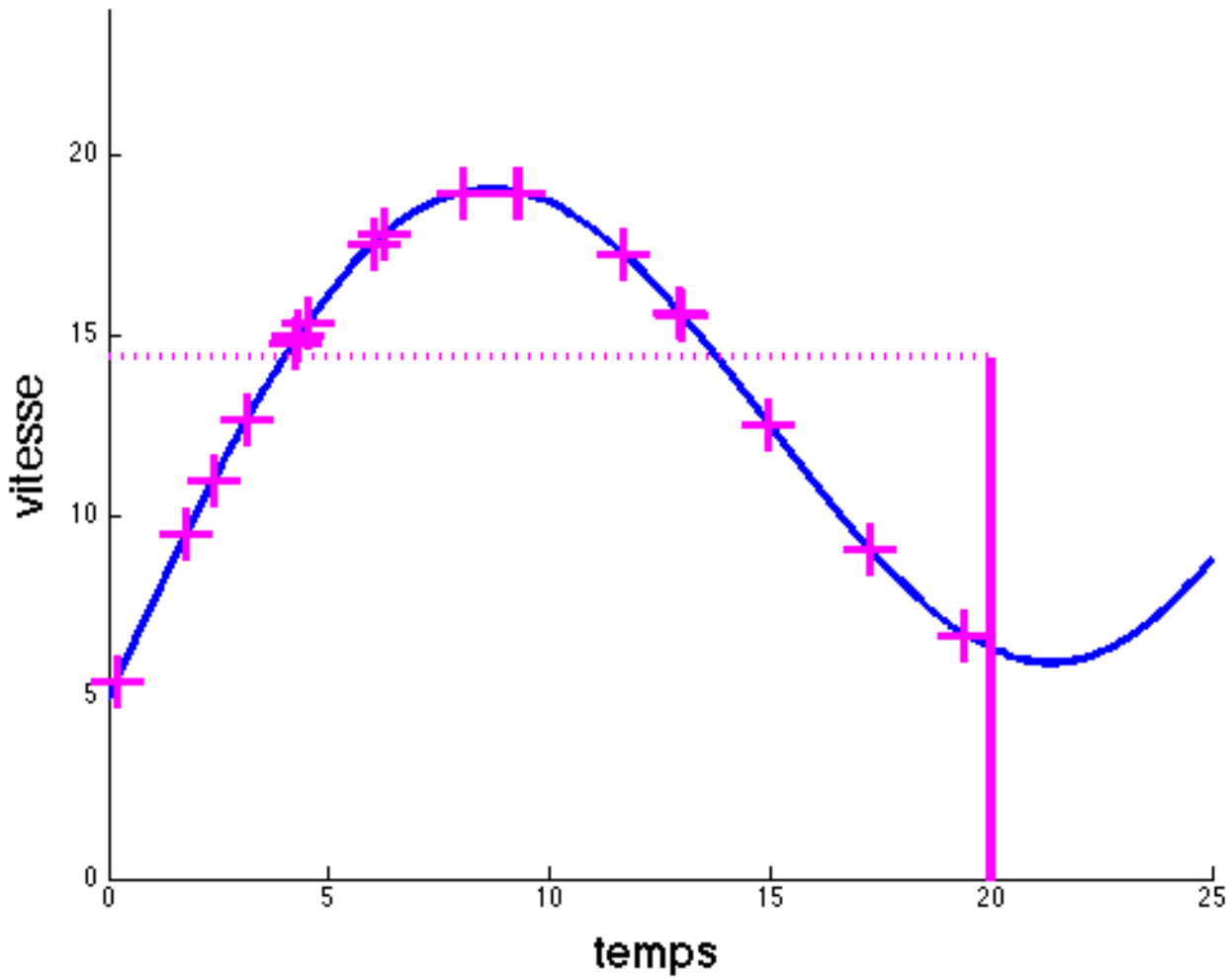
La méthode Monte Carlo



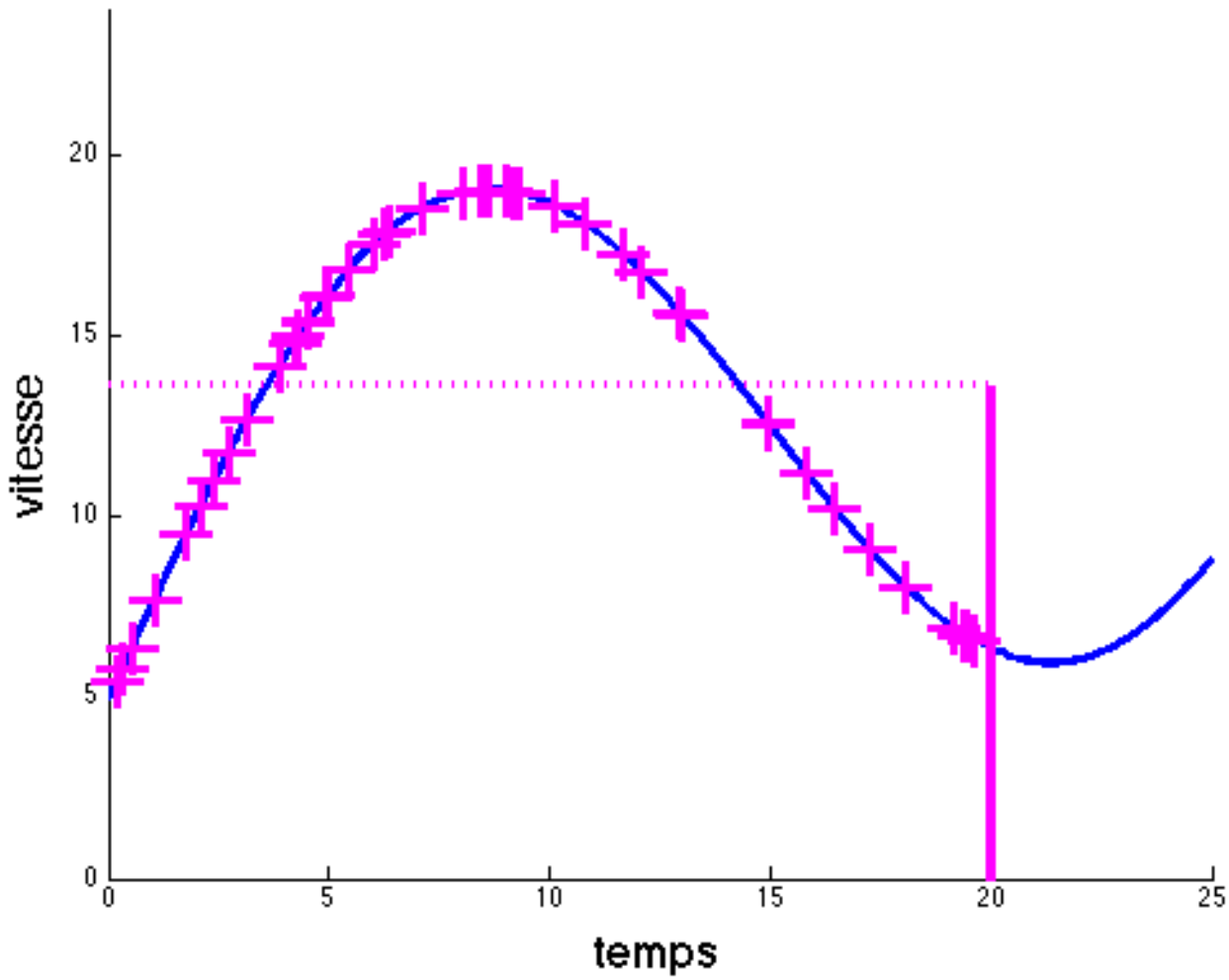
La méthode Monte Carlo



La méthode Monte Carlo



La méthode Monte Carlo



Les équations aux dérivées partielles

Equation de la chaleur.

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2}$$

Mécanique des fluides.

Modèles de population,
concentration d'espèces chimiques.

Finance.

Recherche en méthodes numériques

Algorithmes, méthodes pour :

- résoudre numériquement de nouveaux problèmes,
- calculer plus vite et avec plus de précision.

Un exemple :

Calculez $3^8 = 3*3*3*3*3*3*3*3$.

To boldly go where no human has gone before …

Se poser des questions … auxquelles personne n'a la réponse.

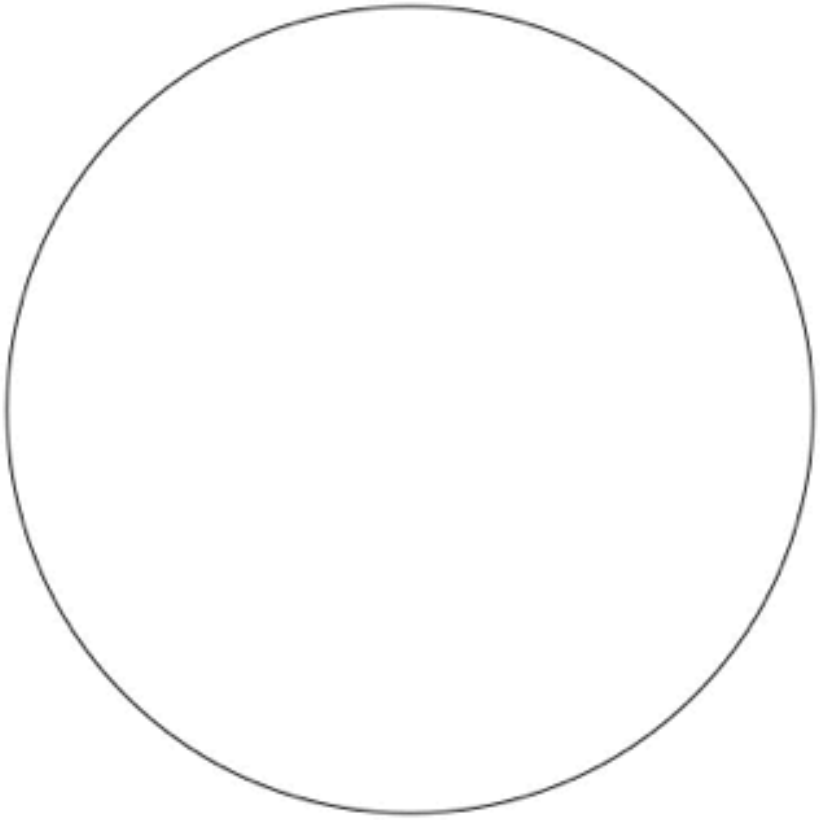
chercher sécher arrêter attendre reprendre analyser imaginer concevoir définir gribouiller essayer calculer éliminer […] regarder voir

biffer soupirer jeter insister changer crayonner traiter examiner comparer […] dériver s'étonner vérifier repérer barrer […] renoncer oublier […] sortir observer classer tourner marmonner retourner considérer rapprocher comprendre inventer adapter […] résoudre intégrer optimiser bricoler […] interpréter trouver sourire tester jubiler consolider énoncer démontrer noter rédiger

Michèle Audin, *Infinitifs* (sur Images des Mathématiques)

Explorer au delà du déjà-connu

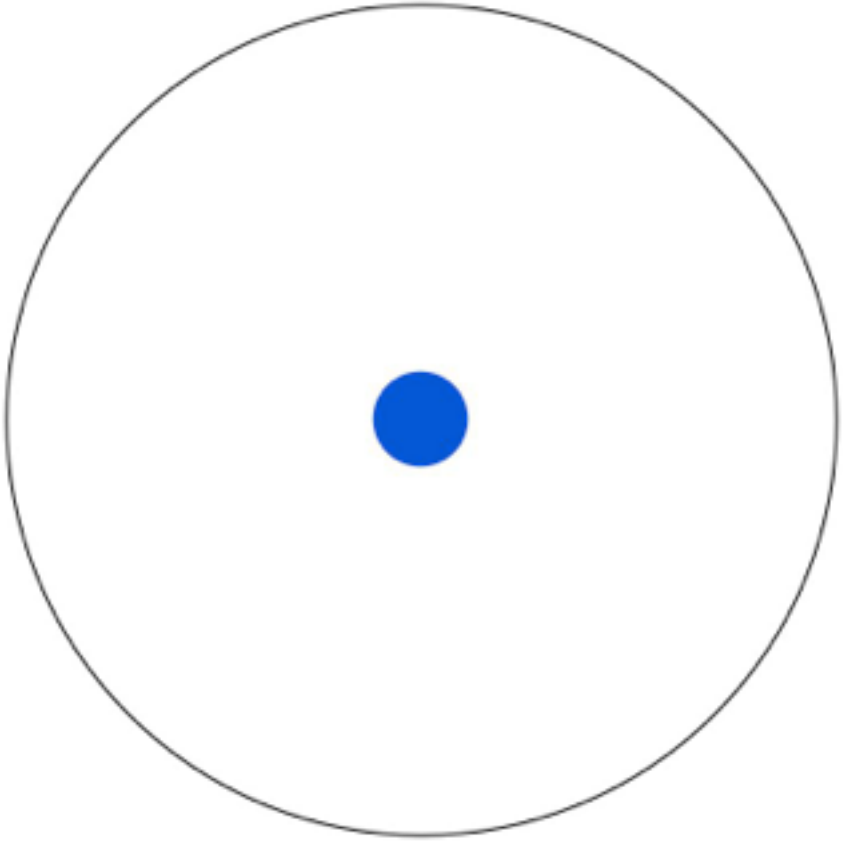
Imagine a circle that contains all of human knowledge:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

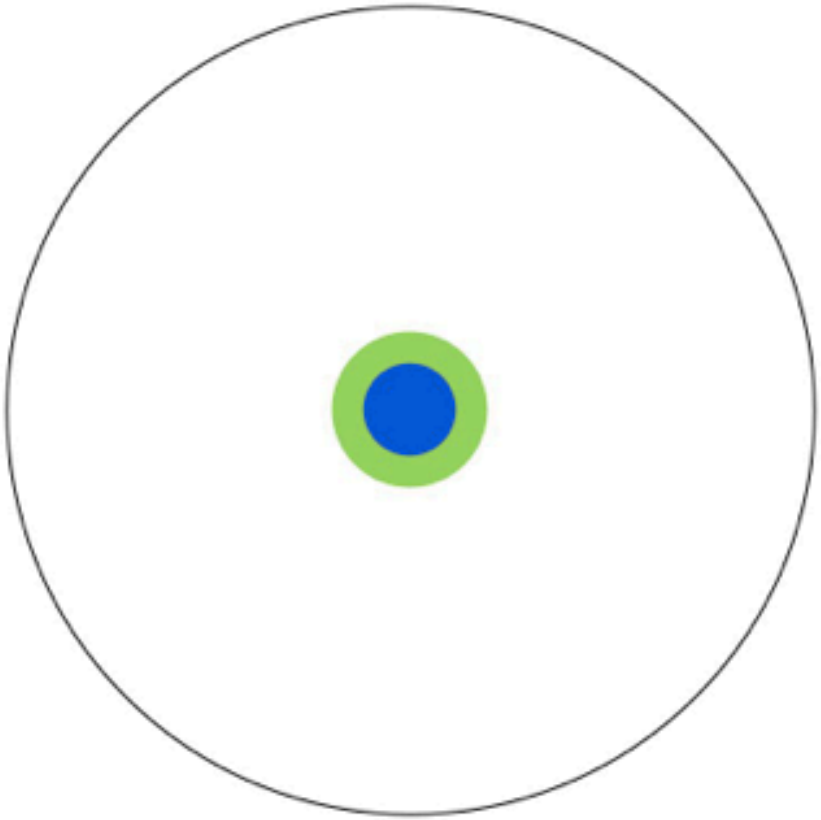
By the time you finish elementary school, you know a little:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

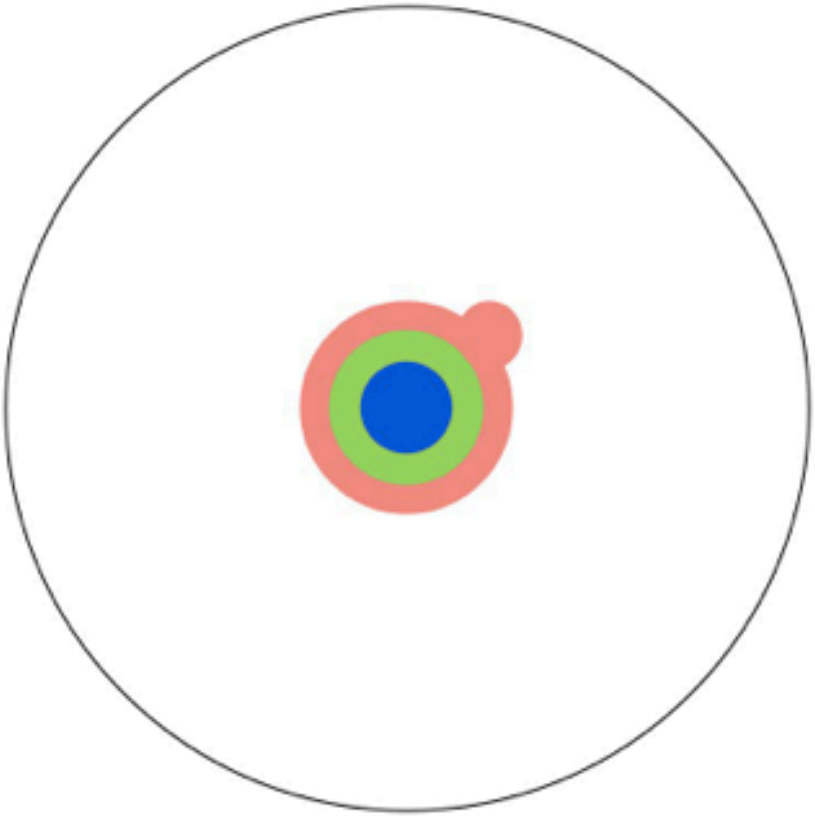
By the time you finish high school, you know a bit more:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

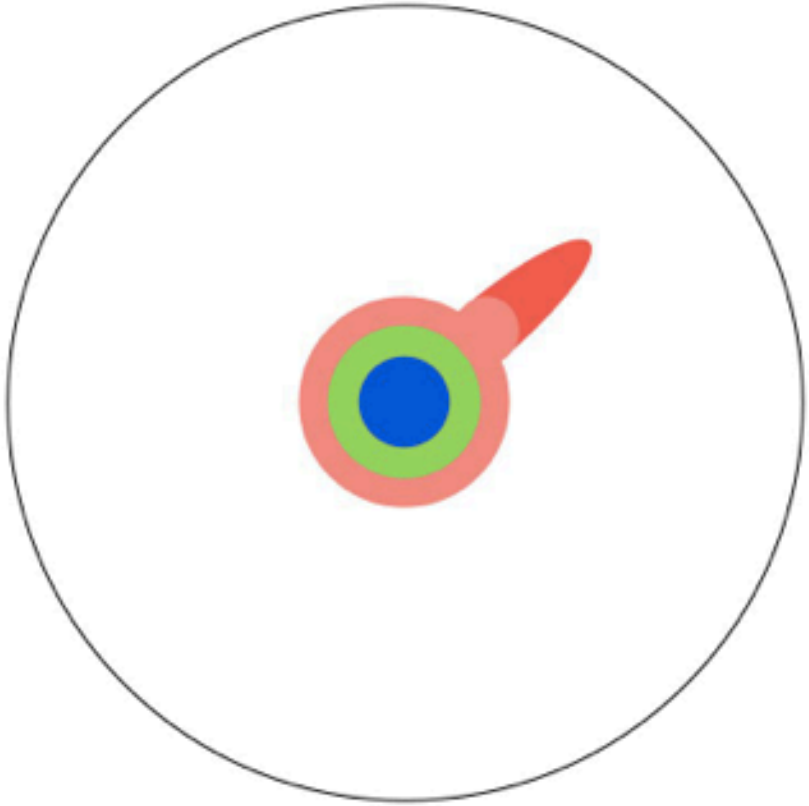
With a bachelor's degree, you gain a specialty:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

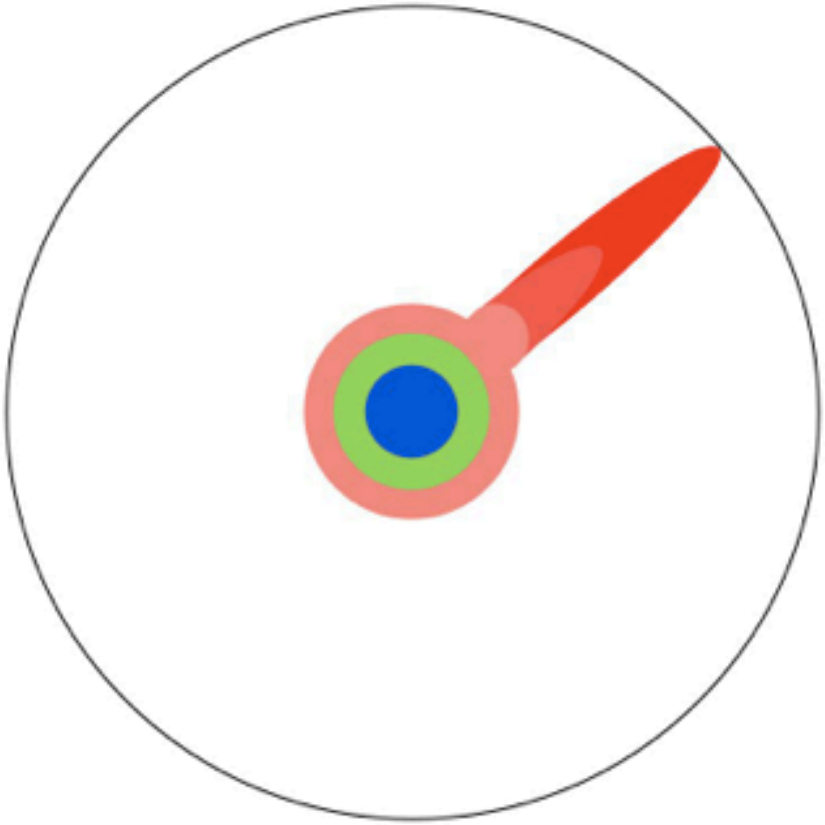
A master's degree deepens that specialty:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

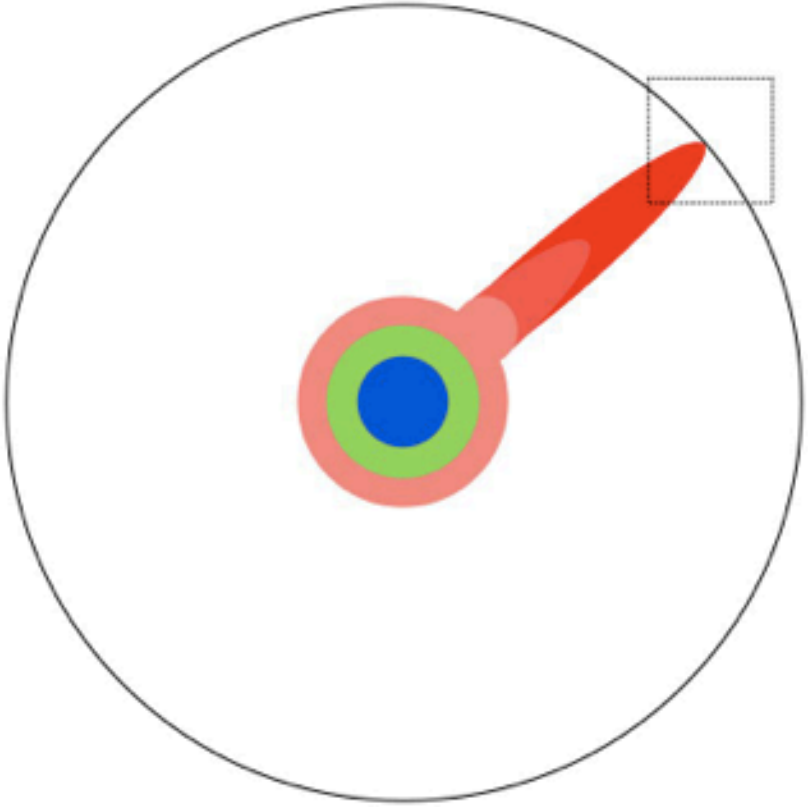
Reading research papers takes you to the edge of human knowledge:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

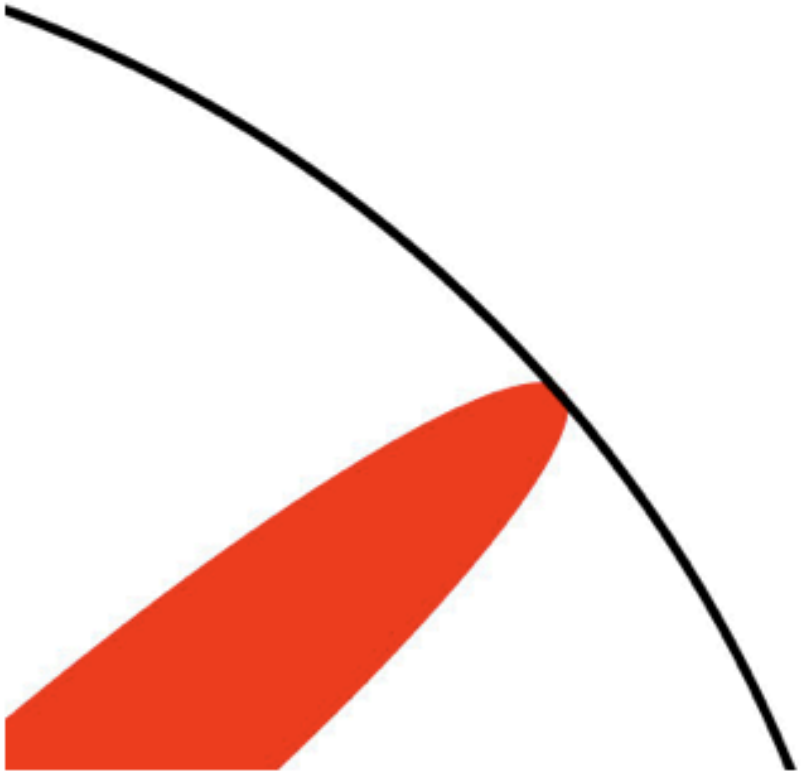
Once you're at the boundary, you focus:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

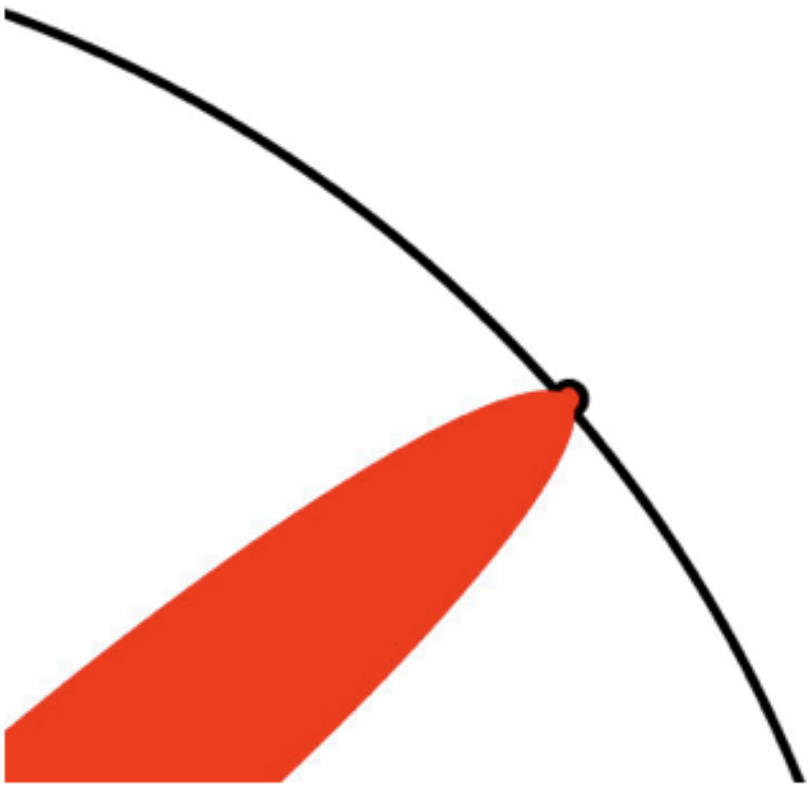
You push at the boundary for a few years:



© Matt Might (http://matt.might.net/)

Explorer au delà du déjà-connu

Until one day, the boundary gives way:



© Matt Might (http://matt.might.net/)

That's all Folks!