Network of Queues with Inert Customers and Signals

J.M. Fourneau

Laboratoire PRiSM, CNRS UMR 8144

Université de Versailles St-Quentin

Joint work with Thu Ha Dao Thi (PRiSM, Univ. Versailles) and Min Anh Tran (LACL, Univ. Creteil)

Basic Ideas

- Queues with customers and signals
- Several Classes of customers
- Symmetric Queues
- Inert Customers
- Quasi-reversibility
- Product Form Steady-State Distribution

Symmetric Queues (Kelly)

- Let n be the number of customers in the queue. $\mu(n)$ is the service capacity.
- A proportion $\gamma(l, n)$ of the total service effort is directed to customer in position l $(1 \leq l \leq n)$. When its service is completed, customers in positions $l+1, l+2, \cdots, n$ move to positions $l, l+1, \cdots, n-1$.
- Upon arrival, a customer moves to position l $(1 \leq l \leq n+1)$ with probability $\gamma(l, n+1)$. Customers previously in positions $l, l+1, \cdots, n$ move respectively to positions $l+1, l+2, \cdots, n+1$.
- γ: proportional function. determines the service discipline.
- $\sum_{l \leq n} \gamma(l, n) = 1$.
- Symmetric: same function γ.
- PS, LIFO and IS are symmetric disciplines. FIFO is not.

Networks of Queues with Signals and Customers

- At the completion of its service, a customer can be routed as a customer or as a signal.
- The interaction between a customer and a signal has an effect and it may create another signal which is routed in the network.
- Complex Interaction between queues.
- We only consider the following signals:
  - negative customers: deletion of a positive customer if any
  - negative signal: deletion of a positive customer and propagation of another negative signal in the same queue or in a twin queue.

Symmetric Queues with Signals

- Upon arrival, a signal will chose a customer in position l as a "target" with probability $\gamma(l, n)$. The signal may succeed or fail to delete the targeted customer depending of its class. The target customer is cancelled with a probability depending on the class.
- Again same function γ.
- To be consistent with signals, $\mu(n) = 1, \quad \forall n$.
- LIFO and PS are consistent with signals, Infinite Server (IS) is not.

Inert Customers

- A customer interacts with server (service rate) and signals (arrival rate of signal, probability of success for the interaction)
- If this probability is 0, no interaction between signals and customers.
- Duality: a new type of customers which are not served ($\mu = 0$) but which interact with the signals
- Inert Customer: no interaction with server, probability of success for the interaction with signal $= 1$.
- Here: only one class of inert customer. Possible to add several classes with distinct routing matrices.
- Waste of service capacity. Blocking.

Positive service rates-Assumptions

- Poisson arrivals of customers and signals from the outside
- Exponential Service
- Markov routing
- Independence
- State of the queue = ordered list of customers (i.e. their class)
- Theorem : The queue is quasi-reversible (under some conditions on the flow equations)
- Proof in Computer Journal 2011.

Assume that the service rates are zero

- But we still have interactions with signals.
- But we must have "well defined networks" to be sure that the interactions between customers and signals take place.
- We obtain inert customers
- Are the previous result (i.e.. product form) still valid ?

The answer is YES

Theorem 2: Consider a queue with inert customers as above. If

$$\rho_c = \frac{\lambda_c}{\mu_c + \lambda^- q_c} < 1, \quad (usual\ customers) \tag{1}$$

$$\rho_c = \frac{\lambda_c}{\lambda^- q_c} < 1, \quad (inert\ customers) \tag{2}$$

then the queue is stable and the steady state is given by

$$\pi(x(1), \cdots, x(n)) = C \prod_{l=1}^{n} \rho_{x(l)}. \tag{3}$$

Furthermore the queue is quasi-reversible in sense defined by Chao. And networks of quasi-reversible queues with Markov routing for signals and customers have product form

Some examples

- Queues and Tanks (stock, counters)
- LIFO queues and iterated deletion of inert customers (blocking)
- PS queues with a controller (a part of the service capacity is waste)

Queues and Tanks

- A tank is a queue which only receives inert customers.
- The total service capacity is wasted.
- One class of usual customers in the network (to simplify)
- Inert customers are sent by other queues and moved (or deleted) by signals.
- Some assumptions on the routing matrices.
- $\mathcal{Q}$ is the set of queues and $\mathcal{T}$ the set of tanks.

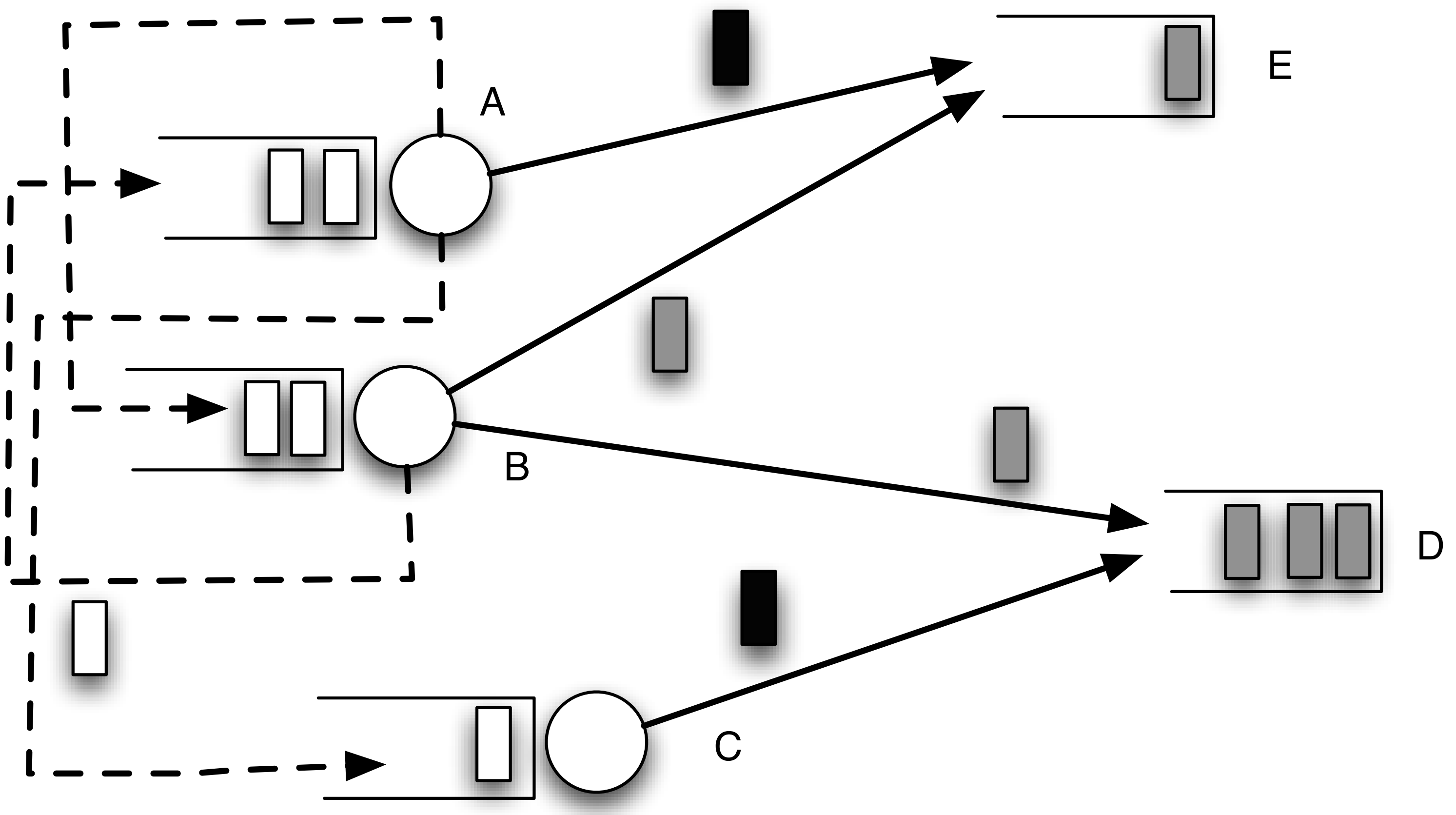

Figure 1: An example of queues and tanks

Solution

Corollary: Assume that the Markov chain of the network with queues and tanks is ergodic. Let ρ_i be the solution of the flow equation:

$$\rho_i = \frac{\lambda_i + \sum_{j \in \mathcal{Q}} \mu_j \rho_j p^{j,i}}{\mu_i + \lambda^{i,-}} \quad if \quad i \in \mathcal{Q},$$

and

$$\rho_i = \frac{\sum_{j \in \mathcal{Q}} \mu_j \rho_j p_s^{j,i}}{\sum_{j \in \mathcal{Q}} \mu_j \rho_j p_i^{j,i}} \quad if \quad i \in \mathcal{T}.$$

If for all i, the inequality $\rho_i < 1$ hold, then the network has a product form steady-state distribution:

$$\pi(x) = \prod_{i \in \mathcal{Q}} (1 - \rho_i) \rho_i^{x_i} \prod_{j \in \mathcal{T}} (1 - \rho_j) \rho_j^{x_j}.$$

Synchonized Deletions

- Tanks are paired during the interaction with signals.
- in the example, A and B are ordinary queues while C and D are tanks. The pair C, D is ordered. item At its arrival in C, the signal deletes an inert customer in C if there is any and jumps to tank D as a signal. If tank C is empty at its arrival, the signals disappears.
- The signal arriving in tank D proceeds in almost the same manner. If tank D is empty at its arrival, the signals disappears. Otherwise, the signal deletes an inert customer in D and jumps to tank C as a signal.
- The iterated deletion of inert customers in tank C and D continue until the smallest queue becomes empty.
- Instantaneous deletion of customers in two tanks

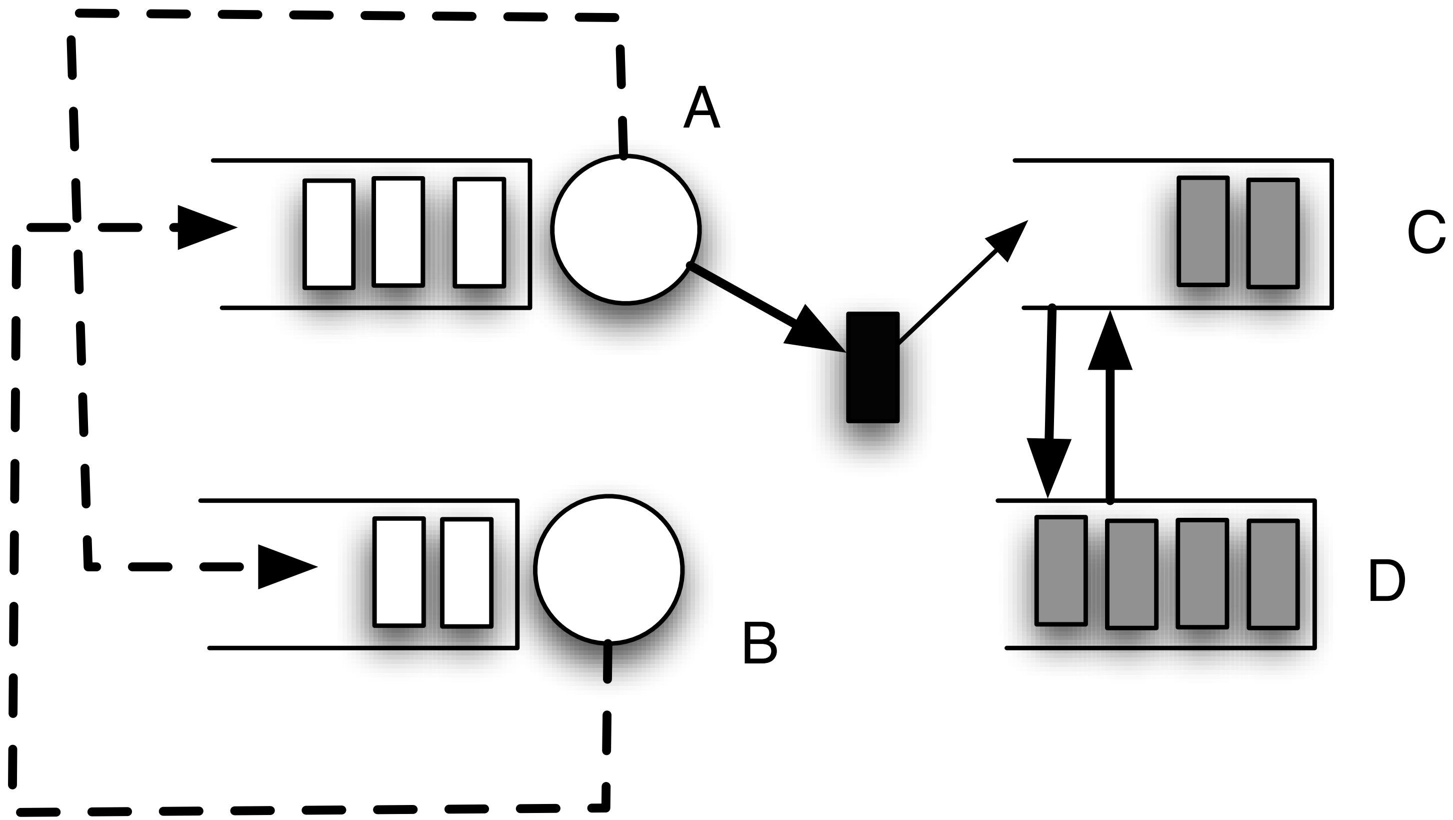

Figure 2: Synchronized deletions - before arrival of signal

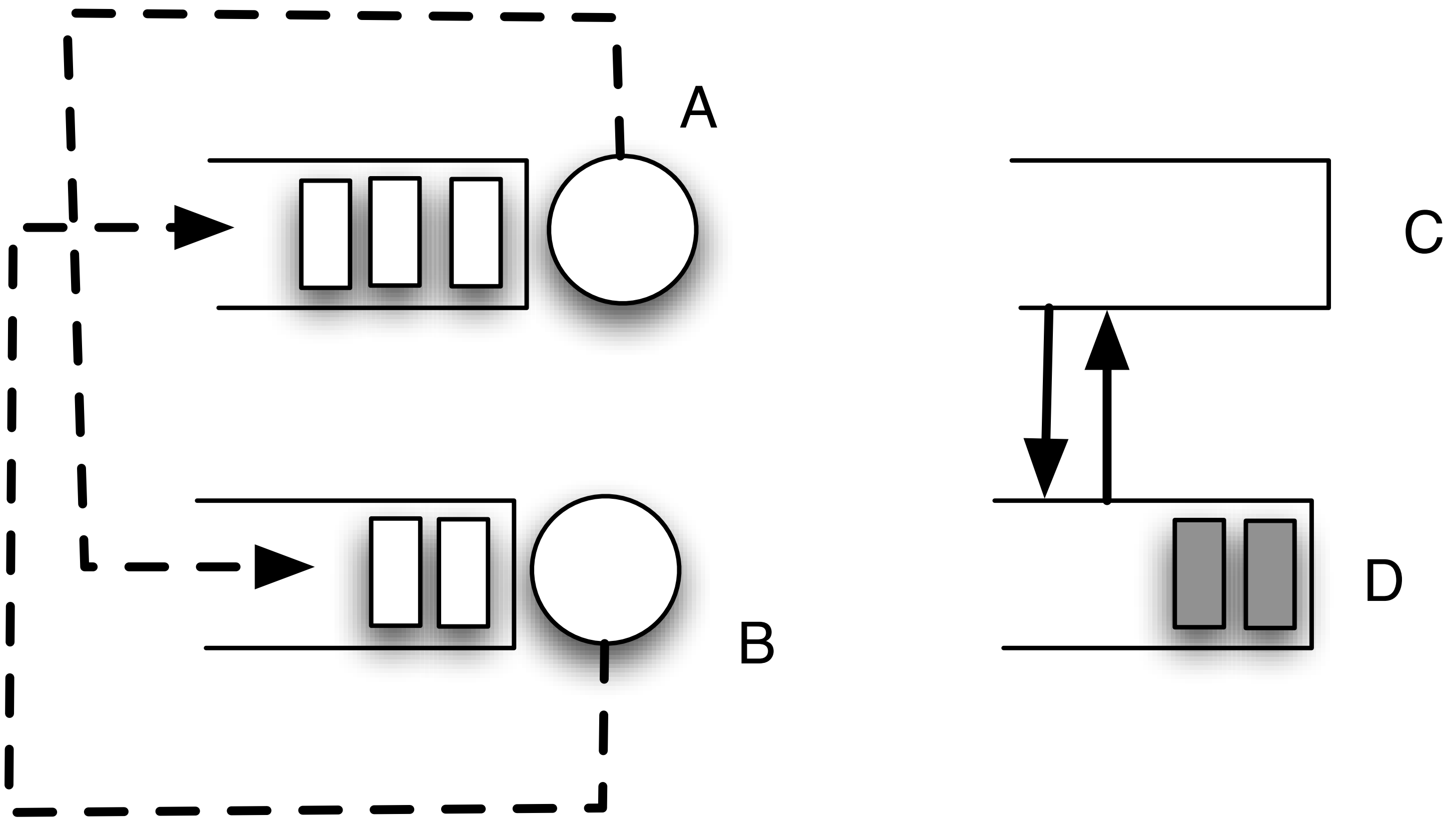

Figure 3: Synchronized deletions - after arrival of signal

Solution

Corollary: We assume ergodicity. If there exists a solution to the flow equation

$$\rho^A = \frac{\lambda^A + \mu^B \rho^B p^{B,A}}{\mu^A}, \qquad \rho^B = \frac{\lambda^B + \mu^A \rho^A p^{A,B}}{\mu^B},$$

$$\rho^C = \frac{\mu^A \rho^A p^{A,C} + \mu^B \rho^B p^{B,C}}{\frac{\mu^A \rho^A p_-^{A,C}}{1 - \rho^C \rho^D}},$$

$$\rho^D = \frac{\mu^A \rho^A p^{A,D} + \mu^B \rho^B p^{B,D}}{\frac{\mu^A \rho^A p_-^{A,C} \rho^C}{1 - \rho^C \rho^D}},$$

such that $\rho_i < 1$ for all i, then the steady-state has a product form distribution:

$$\pi(x) = \prod_{i \in \{A,B,C,D\}} (1 - \rho_i) \rho_i^{x_i}.$$

LIFO queue

- Assuming that a signal deletes inert customers almost surely, but fails to delete an usual one.
- Furthermore after deleting an inert customers, the signal loops.
- When an inert customers is in service, the service capacity of the station is completely wasted
- May stop: because of the arrival of a new usual customer (LIFO) or a signal which cancels the inert customers
- To represent failures ?
- A link with restart strategy ?
- Solution in the paper.

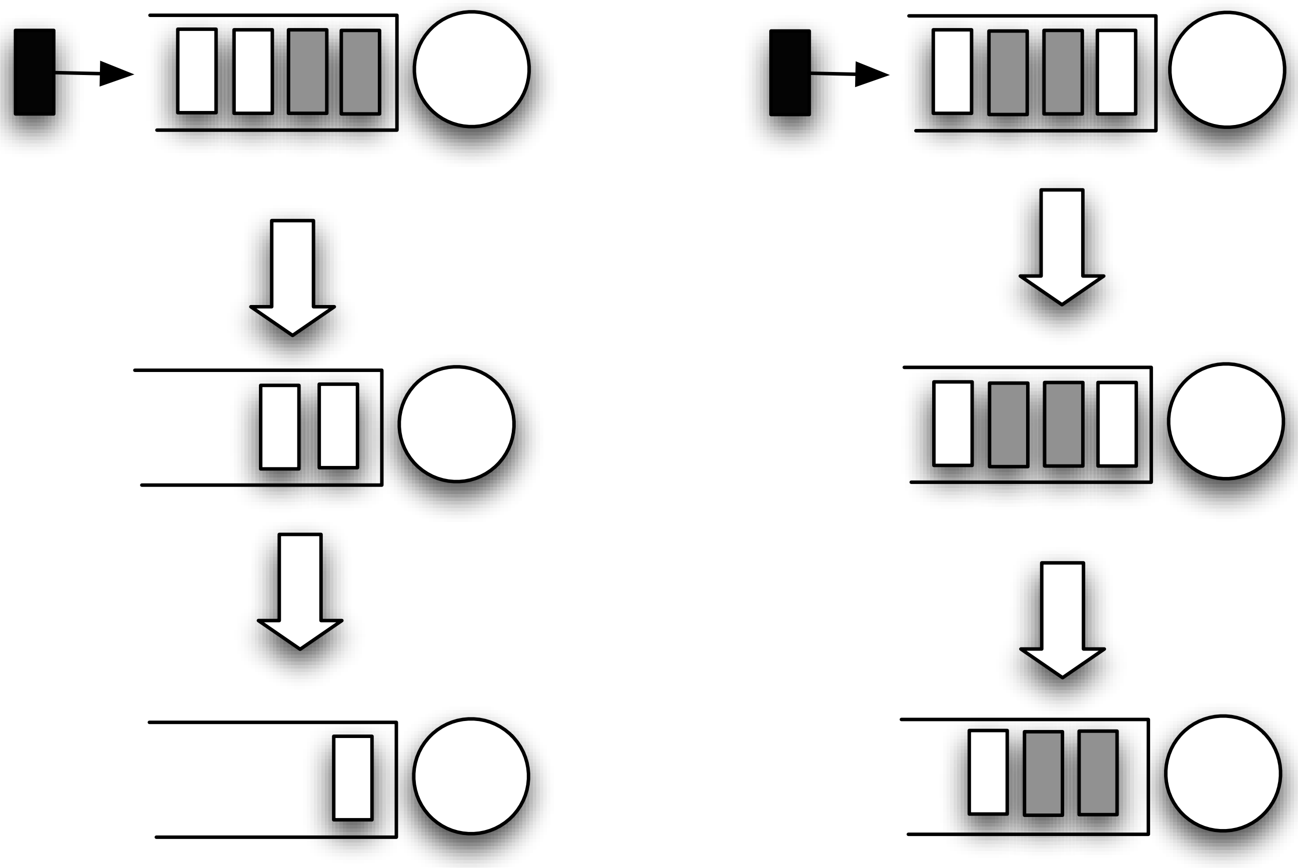

Figure 4: Two sample paths for a LIFO queue for an arrival of signal followed by a service.

Conclusion

- Add more classes of inert customers with distinct routing matrices (like with Kelly networks) after interaction
- Allows a more detailed interaction between signals and customers.
- Check if previous results with multiple classes of customers also apply with $\mu_c = 0$ for some classes.
- Same idea for Petri nets with signals ?