Transition probabilities with service

The transition probabilities for the queue with:

- batch arrivals with probabilities $a_k = \mathbb{P}\{A = k\}$
- batch services of size B
- finite capacity K

The process $\{R_n\}$ is that of the number of customers in the queue at slot n after the events

- service completions
- admitting waiting customers in service
- departure of impatient customers
- arrival of new customers

in this order.

Transition probabilities with service

The evolution equation is:

$$R_{n+1} \; = \; S([R_n - B]^+) + A_n$$

where $S(k)$ has a Binomial distribution with parameters k and $1 - \alpha$.

For $i \leq B$ and $0 \leq j < K$,

$$P_{ij} = a_j \; .$$

For $i \leq B$ and $j = K$,

$$P_{ij} = \sum_{j=K+1}^{\infty} a_j \; .$$

For $i \geq B$ and $i - B \leq j < K$

$$P_{ij} = \sum_{z=0}^{j} a_z \binom{i-B}{j-z} \alpha^{i-B-(j-z)} (1-\alpha)^{j-z} \ .$$

For $i \geq B$ and $j = K$

$$P_{ij} = \sum_{z=0}^{j} a_z \sum_{\ell=K}^{\infty} \binom{i-B}{\ell-z} \alpha^{i-B-(\ell-z)} (1-\alpha)^{\ell-z} \ .$$

Cost formulas

For each $0 \le x \le K$ and $q \in \{0, 1\}$,

$$\begin{aligned} c(x, q) &= q\, c_B + (c_L\, \alpha + c_H)\, (x - qB)^+ \\ &= q\, c_B + c_Q\, (x - qB)^+ \ , \end{aligned}$$

with $c_Q = c_L\, \alpha + c_H$.

Transition probabilities without service

When there is no service, the evolution equation is:

$$R_{n+1} \; = \; S(R_n) + A_n$$

where $S(k)$ has a Binomial distribution with parameters k and $1 - \alpha$.

For $i = 0$ and $0 \leq j < K$,

$$P_{ij} = a_j \ .$$

For $i \leq B$ and $j = K$,

$$P_{ij} = \sum_{j=K+1}^{\infty} a_j \ .$$

For $i \geq 0$ and $0 \leq j < K$

$$P_{ij} = \sum_{z=0}^{j} a_z \binom{i}{j-z} \alpha^{i-(j-z)} (1-\alpha)^{j-z} \ .$$

For $i \geq 0$ and $j = K$

$$P_{ij} = \sum_{z=0}^{j} a_z \sum_{\ell=K}^{\infty} \binom{i}{\ell-z} \alpha^{i-(\ell-z)} (1-\alpha)^{\ell-z} \ .$$

Cost formulas

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