Bounds for queueing networks under histogram-based traffic models

Farah Ait Salaht [1] Hind Castel-Taleb[2] Jean-Michel Fourneau [1] Nihal Pekergin [3]

[1]PRISM, UVSQ

[2]INSTITUT TELECOM, Telecom SudParis

[3]LACL, Université Paris-Est Val de Marne

Motivation

- Traffic characterization :
  - Exponential arrivals and services, mathematical formulas (example : Erlang Formula)
  - General traffic: Pareto, Weibull : accurate model, high number of parameters, intractable model
  - Exact traffic traces : Histogram based approach, Markov chains with huge state space
- Markov chain Analysis :
  - Stochastic bounds to reduce the size of the probability distributions
  - Bounds on performance mesures
  - Control the distribution size, and the accuracy of the results

Outline

- Stochastic ordering theory
- Optimal stochastic bounds algorithm
- Queueing models with histogram traffic
    - Stochastic Monotonicity proofs
    - Performance measure bounds : numerical results
- Queueing network analysis (DAG)
    - Monotonicity proofs for networking elements : merge, split
    - Numerical results for a queueing system
- Conclusion

Outline

Bounds, Stochastic comparisons

Definition

$$X \leq_{st} Y \Longleftrightarrow \mathbb{E}f(X) \leq \mathbb{E}f(Y)$$

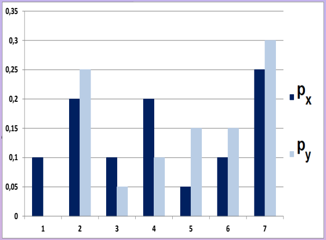
for all non decreasing functions $f : \mathbb{G} \rightarrow \mathbb{R}^+$ whenever expectations exist.

Proposition

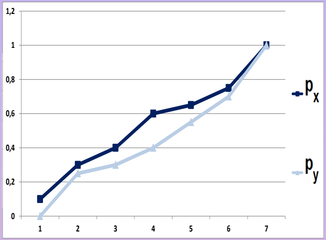
$$X \leq_{st} Y \Leftrightarrow \forall i,\ 1 \leq i \leq n,\ \sum_{k=i}^{n} d2(k) \leq \sum_{k=i}^{n} d1(k) \qquad (1)$$

Stochastic bounds

Example: We consider $\mathcal{G} = \{1, 2, \ldots, 7\}$,
$\mathbf{p}_X = [0.1,\ 0.2,\ 0.1,\ 0.2,\ 0.05,\ 0.1,\ 0.25]$ and
$\mathbf{p}_Y = [0,\ 0.25,\ 0.05,\ 0.1,\ 0.15,\ 0.15,\ 0.3]$.



The pmf of a discrete distributions X and Y



Cumulative distribution functions

Outline

Bounding probability distributions

For a given distribution d of size N, and a measure $\sum_{i \in \mathbb{E}} r(i)d(i)$ $(r : \mathbb{E} \to \mathbb{R}^+)$,
$\Rightarrow$ **we compute bounding distributions** $d1$ **and** $d2$ **of size** $K < N$ such that:

1. $d2 \leq_{st} d \leq_{st} d1$,
2. $\sum_{i \in \mathbb{E}} r(i)d(i) - \sum_{i \in \mathbb{E}^l} r(i)d2(i)$ is minimal among the set of distributions on n states that are stochastically lower than d,
3. $\sum_{i \in \mathbb{E}^u} r(i)d1(i) - \sum_{i \in \mathbb{E}} r(i)d(i)$ is minimal among the set of distributions on n states that are stochastically upper than d.

Example

Let :

- $\mathbb{E} = \{1, 2, 3, 4, 5, 6\}$,
- $\mathbf{r}[i] = i$,
- $\mathbf{d} = [0.3, 0.1, 0.1, 0.1, 0.2, 0.2]$. The initial accumulated reward is $R = 3.4$.

If we remove states 2, 3, 6, we found :

- $\mathbf{d} = [0.5, 0.1, 0.4]$ is a lower bound of $\mathbf{d}$
- on state space $\mathbb{F} = \{1, 4, 5\}$,
- and the reward is equal to 2.9.

Bounding histogram reduction

Optimal Algorithm based on dynamic programming

- Graph theory problem.
- Consider the weighted graph $G = (V, E)$ with:
  - **Lower Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(j) - \mathbf{r}(u))$
  - **Upper Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(v) - \mathbf{r}(j))$

Computing optimal bound $\equiv$ Compute a shortest path in graph G with K nodes ($K << N$).

- A mass probability of removed nodes is summed with the
  - **Lower Bound:** immediate predecessor
  - **Upper Bound:** immediate successor

Complexity: $O(N^2 K)$ and cubic when K has the same order as N.

Bounding histogram reduction

Optimal Algorithm based on dynamic programming

- Graph theory problem.
- Consider the weighted graph $G = (V, E)$ with:
  - **Lower Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(j) - \mathbf{r}(u))$
  - **Upper Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(v) - \mathbf{r}(j))$

Computing optimal bound $\equiv$ Compute a shortest path in graph G with K nodes ($K << N$).

- A mass probability of removed nodes is summed with the
  - **Lower Bound:** immediate predecessor
  - **Upper Bound:** immediate successor

Complexity: $O(N^2 K)$ and cubic when K has the same order as N.

Bounding histogram reduction

Optimal Algorithm based on dynamic programming

- Graph theory problem.
- Consider the weighted graph $G = (V, E)$ with:
  - **Lower Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(j) - \mathbf{r}(u))$
  - **Upper Bound:** $w(e) = \sum_{j \in \mathcal{H}: u<j<v} \mathbf{d}(j)(\mathbf{r}(v) - \mathbf{r}(j))$

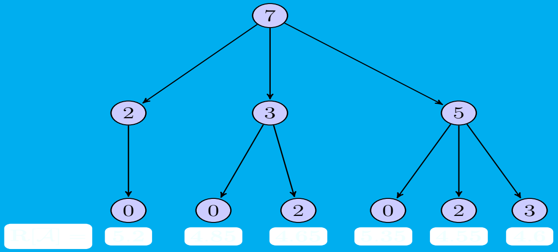
Computing optimal bound $\equiv$ Compute a shortest path in graph G with K nodes ($K << N$).

- A mass probability of removed nodes is summed with the
  - **Lower Bound:** immediate predecessor
  - **Upper Bound:** immediate successor

Complexity: $O(N^2 K)$ and cubic when K has the same order as N.

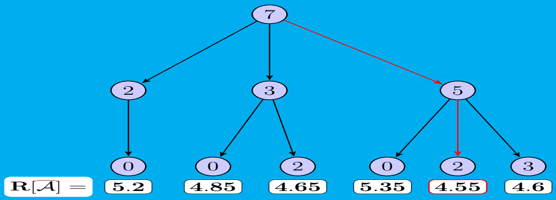
Example: Optimal upper bound

- Discrete distribution $\mathcal{A} = (\mathbf{A}, p(\mathbf{A}))$ with $\mathbf{A} = \{0, 2, 3, 5, 7\}$ and $p(\mathbf{A}) = [0.05, 0.3, 0.15, 0.2, 0.3]$, Reward function $\mathbf{r}$: $\forall\, a_i \in \mathbf{A},\ \mathbf{r}(a_i) = a_i,\ R[\mathcal{A}] = \sum_{a_i \in \mathbf{A}} \mathbf{r}(a_i)\, p_{\mathbf{A}}(i) = 4.15$
- Compute the Optimal Upper Bound $\overline{\mathcal{A}}$ on 3 states such that $R[\overline{\mathcal{A}}] - R[\mathcal{A}]$ is minimal.



Example: Optimal upper bound

- Discrete distribution $\mathcal{A} = (\mathbf{A}, p(\mathbf{A}))$ with $\mathbf{A} = \{0, 2, 3, 5, 7\}$, $p(\mathbf{A}) = [0.05, 0.3, 0.15, 0.2, 0.3]$, Reward function $\mathbf{r}$: $\forall\, a_i \in \mathbf{A}$, $\mathbf{r}(a_i) = a_i$, $R[\mathcal{A}] = \sum_{a_i \in \mathbf{A}} \mathbf{r}(a_i)\, p_{\mathbf{A}}(i) = 4.15$.
- Compute the Optimal Upper Bound $\overline{\mathcal{A}}$ on 3 states such that $R[\overline{\mathcal{A}}] - R[\mathcal{A}]$ is minimal.



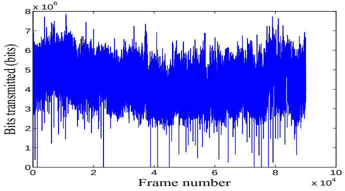
$\overline{\mathcal{A}} = (\overline{\mathbf{A}}, p(\overline{\mathbf{A}}))$ with $\overline{\mathbf{A}} = \{2, 5, 7\}$, $p(\overline{\mathbf{A}}) = [0.35, 0.35, 0.3]$ and $R[\overline{\mathcal{A}}] = 4.55$.

Outline

Outline

Queueing model description

- **Traffic trace used as illustration:**

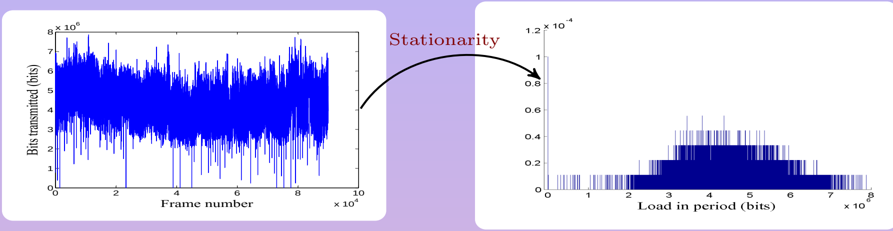


Stationar

- MAWI traffic trace corresponds to a 1-hour IP traffic
  9th of January 2007 between 12 and 13
- Sampling period: $T = 40\,ms$

Queueing model description

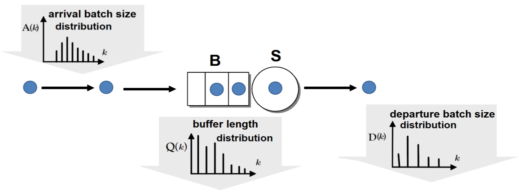
- **Traffic trace used as illustration:**



- MAWI traffic trace corresponds to a 1-hour IP traffic
  9th of January 2007 between 12 and 13
- Sampling period: $T = 40\ ms$
- Histogram representation
- The number of bins is 80511

Queueing model description

- **Queueing model:**



Arrival traffic is stationary and i.i.d. $(A(t) = \mathcal{A})$.

State evolution equations

- The queue length equation:

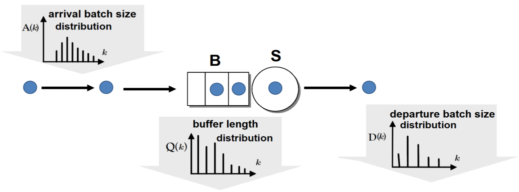
$$Q(k) = \mathtt{min}(B, (Q(k-1) + \mathcal{A} - S)^{+}), \quad k \in \mathbb{N}. \qquad (2)$$

- The departure distribution under Tail Drop policy:

$$D(k) = \mathtt{min}(S, Q(k-1) + \mathcal{A}), \quad k \in \mathbb{N}. \qquad (3)$$

Queueing model description

- **Queueing model:**



Arrival traffic is stationary and i.i.d. $(A(t) = \mathcal{A})$.

State evolution equations

- The queue length equation:

$$Q(k) = \mathtt{min}(B, (Q(k-1) + \mathcal{A} - S)^+), \quad k \in \mathbb{N}. \tag{2}$$

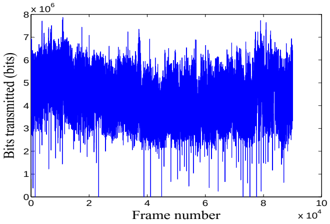
- The departure distribution under Tail Drop policy:

$$D(k) = \mathtt{min}(S, Q(k-1) + \mathcal{A}), \quad k \in \mathbb{N}. \tag{3}$$

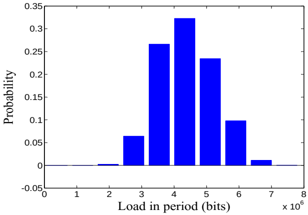
The Histogram Buffer Stochastic Process: HBSP

Presentation of Hernández and al. Model

Goal: Reduce the size of the initial trace $\Longrightarrow$ Accelerate the computation time $\Longrightarrow$ Divide the state space ($|\mathcal{H}| = N$) into K sub-intervals (bins), $K << N$.



MAWI traffic trace

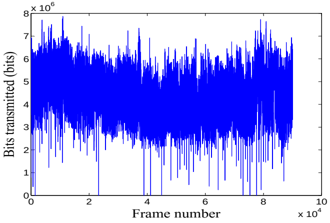


HBSP histogram using K=10 bins

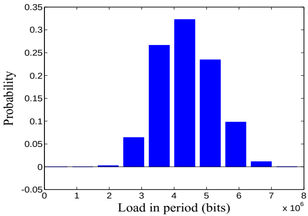
The Histogram Buffer Stochastic Process: HBSP

Presentation of Hernández and al. Model

Goal: Reduce the size of the initial trace $\Longrightarrow$ Accelerate the computation time. $\Longrightarrow$ Divide the state space ($|\mathcal{H}| = N$) into K sub-intervals (bins), $K << N$.



MAWI traffic trace

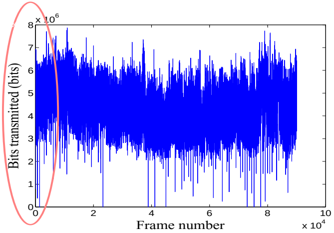


HBSP histogram using K=10 bins

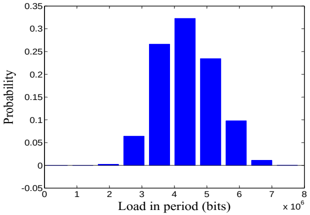
The Histogram Buffer Stochastic Process: HBSP

Presentation of Hernández and al. Model

Goal: Reduce the size of the initial trace $\Longrightarrow$ Accelerate the computation time.$\Longrightarrow$ Divide the state space ($|\mathcal{H}| = N$) into K sub-intervals (bins), $K << N$.



MAWI traffic trace

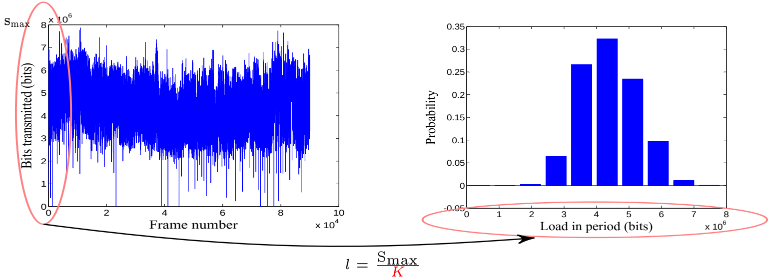


HBSP histogram using K=10 bins

The Histogram Buffer Stochastic Process: HBSP

Presentation of Hernández and al. Model

Goal: Reduce the size of the initial trace $\Longrightarrow$ Accelerate the computation time. $\Longrightarrow$ Divide the state space ($|\mathcal{H}| = N$) into K sub-intervals (bins), $K << N$.

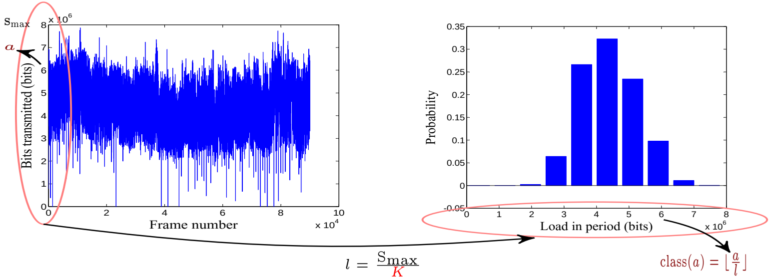


MAWI traffic trace **HBSP histogram using K=10 bins**

The Histogram Buffer Stochastic Process: HBSP

Presentation of Hernández and al. Model

Goal: Reduce the size of the initial trace $\Longrightarrow$ Accelerate the computation time.$\Longrightarrow$ Divide the state space ($|\mathcal{H}| = N$) into K sub-intervals (bins), $K << N$.



MAWI traffic trace **HBSP histogram using K=10 bins**

The Histogram Buffer Stochastic Process: HBSP

Stochastic process: HD/D/1/B queue

Queue length distribution:

$$Q(k) = \min(\hat{B}, (Q(k-1) \otimes \mathcal{A} - \hat{S})^{+}).$$

Where, $\hat{S} = class(S)$, $\hat{B} = class(B)$ and $\otimes$ is the convolution operator of distributions.

Let X and Y two discrete random variables defined on $\mathcal{G}_X$ and $\mathcal{G}_Y$ resp. with $|\mathcal{G}_X| = l_X$ and $|\mathcal{G}_Y| = l_Y$.

Proposition: Convolution complexity

- Convolution of the distributions generates a distribution with at most $l_X \times l_Y$ states.
- And requires: $O(l_X \times l_Y)$ operation $(+)$ using a naive approach;

  $O((l_X + l_Y)log(l_X + l_Y))$ for FFT.

Properties

- An approximative method
- Consider a single node using real workload traces

The Histogram Buffer Stochastic Process: HBSP

Stochastic process: HD/D/1/B queue

Queue length distribution:

$$Q(k) = \min(\hat{B}, (Q(k-1) \otimes \mathcal{A} - \hat{S})^{+}).$$

Where, $\hat{S} = class(S)$, $\hat{B} = class(B)$ and $\otimes$ is the convolution operator of distributions.

Let X and Y two discrete random variables defined on $\mathcal{G}_X$ and $\mathcal{G}_Y$ resp. with $|\mathcal{G}_X| = l_X$ and $|\mathcal{G}_Y| = l_Y$.

Proposition: Convolution complexity

- Convolution of the distributions generates a distribution with at most $l_X \times l_Y$ states.
- And requires: $O(l_X \times l_Y)$ operation $(+)$ using a naive approach;
  $O((l_X + l_Y) log(l_X + l_Y))$ for FFT.

Properties

- An approximative method
- Consider a single node using real workload traces

Outline

Monotonicity results

Goal: Stochastic bound on arrival process $\Longrightarrow$ bound on performance measures.

We prove the following major results:

$$\text{If } A(k) \leq_{st} A^U(k), \forall k \geq 0, \text{ then } Q(k) \leq_{st} Q^U(k), \forall k \geq 0$$

and

$$\text{If } A(k) \leq_{st} A^U(k), \forall k \geq 0, \text{ then } D(k) \leq_{st} D^U(k), \forall k \geq 0.$$

Also true for stationary processes.

Assume that the chain is ergodic and the steady state is π.

$$Q^L(k) \leq_{st} Q^L(k+1) \leq_{st} \pi \leq_{st} Q^U(k+1) \leq_{st} Q^U(k).$$

If $||Q^U(k+1) - Q^L(k+1)||_\infty < \epsilon$ the limit of $Q^L(k)$ and $Q^U(k)$ is π.

Monotonicity results

Goal: Stochastic bound on arrival process $\Longrightarrow$ bound on performance measures.

We prove the following major results:

$$\text{If } A(k) \leq_{st} A^U(k), \forall k \geq 0, \text{ then } Q(k) \leq_{st} Q^U(k), \forall k \geq 0$$

and

$$\text{If } A(k) \leq_{st} A^U(k), \forall k \geq 0, \text{ then } D(k) \leq_{st} D^U(k), \forall k \geq 0.$$

Also true for stationary processes.

Assume that the chain is ergodic and the steady state is π.

$$Q^L(k) \leq_{st} Q^L(k+1) \leq_{st} \pi \leq_{st} Q^U(k+1) \leq_{st} Q^U(k).$$

If $||Q^U(k+1) - Q^L(k+1)||_\infty < \epsilon$ the limit of $Q^L(k)$ and $Q^U(k)$ is π.

Outline

Real traffic experiments

Goal:

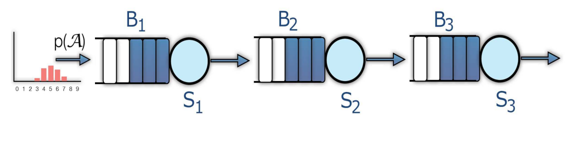
Compare the different methods (Exact result, HBSP method and our bounds).

1- Single Node

- Influence of the number of bins on the accuracy of the results
- Relationship between buffer size and some performance measures

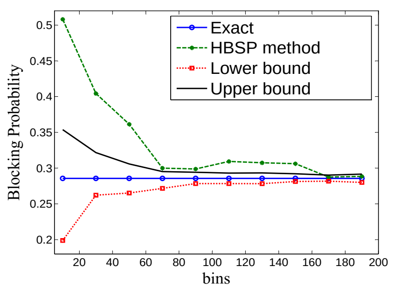
2- Queueing network

We consider the following Tandem queue

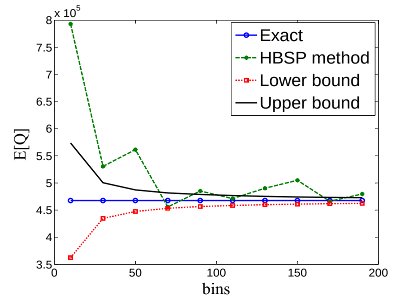


Number of bins vs Accuracy: QoS parameters using MAWI traffic trace

1- Single Node



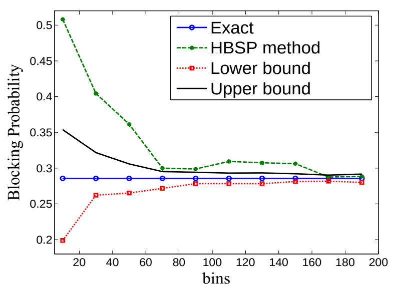
(a) Blocking Probability



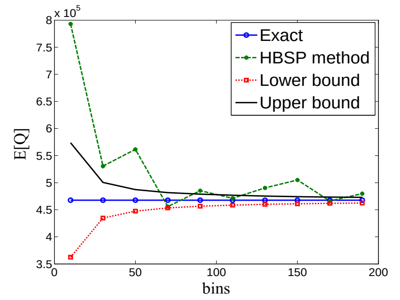
(b) Mean buffer occupancy

Number of bins vs Accuracy: QoS parameters using MAWI traffic trace

1- Single Node



(c) Blocking Probability

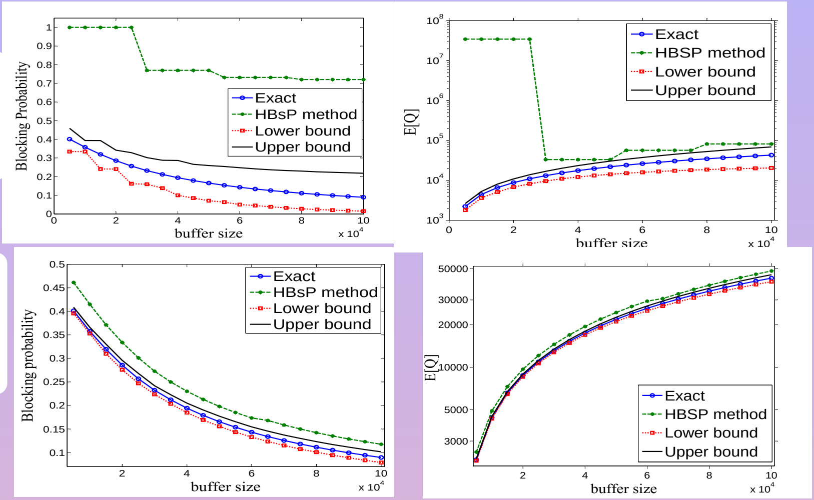


(d) Mean buffer occupancy

QoS parameters using CAIDA OC-48 traffic trace

$bins = 10$

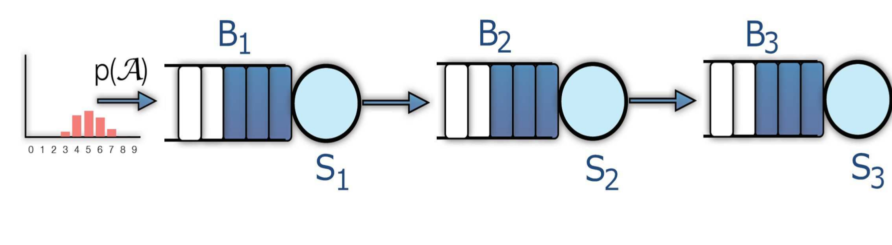
$bins = 100$



Single Node

		Exact	$L.b$	$U.b$	HBSP	Tancrez U. b
Buffer capacity (10^5 bits)	0.5	0.456	0.411	0.469	/	0.491
	2	0.425	0.374	0.437	/	0.464
	3	0.405	0.371	0.437	/	0.460
	4	0.386	0.338	0.407	0.407	0.429
	5	0.367	0.333	0.380	0.407	0.429
	8	0.317	0.271	0.354	0.326	0.372
	9	0.301	0.241	0.330	0.326	0.349
	10	0.285	0.238	0.325	0.326	0.347
	20	0.182	0.124	0.235	0.155	0.258
	30	0.131	0.076	0.173	0.102	0.215
	& Ex. Time (s)	3378	4.5	5.3	0.01	0.06

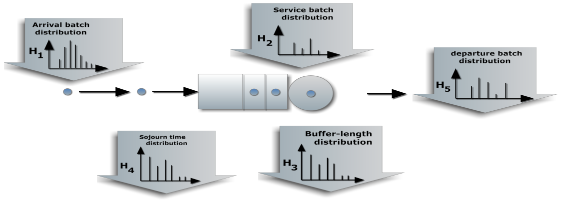
Queueing network: Tandem Queue



Bounds for each queue

- Each queue is analyzed separately
- Monotonicity $\Longrightarrow$ bound on each intermediate stage
- Performance measures : blocking probabilities, Queue length, response ime

H-Monotonicity approach



Theorem

[H-monotonicity for H_3 and H_5] If $H_1^a \leq_{st} H_1^b$ and $H_2^a \geq_{st} H_2^b \implies H_3^a \leq_{st} H_3^b$ and ${}^{a}_{5} \leq_{st} H_5^b$

Theorem (H-Monotonicity for losses)

If $H_1^a \leq_{st} H_1^b$ and $H_2^b \leq_{st} H_2^a$ and the element is work conserving and operated under the Tail Drop policy, then $H_L^a \leq_{st} H_L^b$.

Outline

More general queueing networks, with different flows

- Feed-Forward networks (DAG)
- Decomposition approach, sequential study

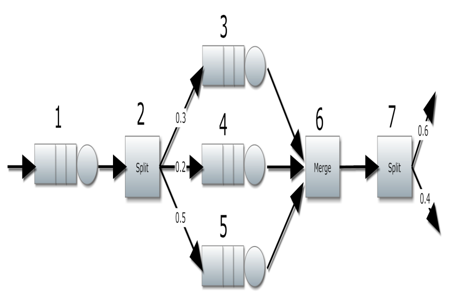


Figure 1 : A queueing network with $n = 7$ nodes.

Description du split et du Merge

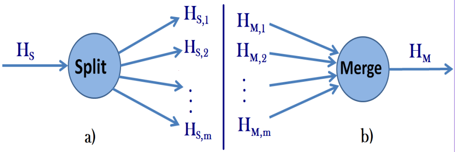


Figure 2 : Split and Merge operations

Monotonicity of Split

Definition

A split is said to be H-monotone, iff
$H_S^a \leq_{st} H_S^b \Rightarrow \forall i,\ H_{S,i}^a \leq_{st} H_{S,i}^b$.

Let $p_i, 1 \leq i \leq m$ (such that $\sum_{i=1}^m p_i = 1$), be the routing probability of the batch to output i of the splitter. The probability distribution of any output flow i is for all $i \leq m$:

$$H_{S,i}(k) = p_i\ H_S(k),\ k \in E^{H_S},\ k > 0$$

$$H_{S,i}(0) = 1 - \sum_{k \neq 0} H_{S,i}(k).$$

Theorem

With a batch routing, the splitter is H-monotone.

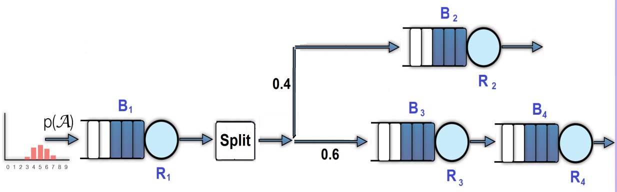


Figure 3 : A tree network with five nodes.

		$\mathbb{E}[H_3]$	$\mathbb{E}[H_5]$	$\mathbb{E}[H_4^l]$	$\mathbb{E}[H_4^u]$	P_L
Queue 1	$L.b$	4232760	4351000	1.5525	/	0.00504
	$U.b$	4353820	4353930	/	1.5766	0.00545
Queue 2	$L.b$	1081130	1739270	1.1098	/	0.00065
	$U.b$	1084180	1740420	/	1.1102	0.00066
Queue 3	$L.b$	2099190	2600510	1.3656	/	0.00386
	$U.b$	2107100	2602110	/	1.3688	0.00392
Queue 4	$L.b$	2480090	2595280	1.5044	/	0.00191
	$U.b$	2494890	2596770	/	1.5091	0.00195

Table 1 : Bounding Results.

Outline

Conclusion

- A novel approach based on stochastic bounds, to derive optimal bounds :
    - bounding aggregation of histograms
    - relevant for the network dimensionning
    - tradeoff between accuracy and complexity
- current and future work
    - Split with division of batch, Merge element
    - Analysis of some AQM mechanisms

Publications

- Farah Ait Salaht, Hind Castel-Taleb, Jean-Michel Fourneau, Nihal Pekergin,"Stochastic Bounds and Histograms for Network Performance Analysis", EPEW2013, 10th European Workshop on Performance Engineering Ca' Foscari University, Venice, Italy, 16-17 September, 2013.
- Farah Ait Salaht, Hind Castel-Taleb, Jean-Michel Fourneau, Nihal Pekergin, "Une approche combinant bornes stochastiques, traces et histogrammes pour l?analyse de performance des réseaux", Modélisation des Systèmes Réactifs (MSR'13) INRIA Rennes - Bretagne Atlantique, France, 13-15 novembre 2013, publié dans la Revue JESA (Volume 27 nř 1-2-3/2013, Lavoisier).
- F. Ait-Salaht, H. Castel-Taleb, J.M. Fourneau, N. Pekergin,"A bounding histogram approach for network performance analysis", HPCC 2013 : 15th IEEE International Conference on High Performance Computing and Communications, Nov 13, 2013 - Nov 15, 2013 Where Zhangjiajie, China.